The Transmission of Supply Shocks in Different Inflation Regimes*

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Abstract

We show that the pass-through of supply shocks to consumer prices depends on prevailing inflation volatility. Using an agnostic Markov-switching specification, the data selects two regimes of inflation dynamics, mainly characterized by high and low volatility. We identify supply shocks with instrumental variables constructed from outliers in producer prices. These shocks have larger and more persistent effects on downstream prices in the high-volatility regime. By contrast, exogenous splits based on the inflation level or the shock size reveal no meaningful state dependence. The evidence supports a model in which firms optimally invest in price flexibility. In that model, stricter inflation targeting lowers inflation volatility through two channels: it reduces optimal price flexibility——dampening shock pass-through to inflation——additionally to stabilizing demand via the standard channel.

Keywords: Inflation regimes, supply shocks, monetary policy, cost pass-through, producer prices

JEL-Codes: E31, E52, E32

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1 Introduction

Policymakers have, particularly during times of rising inflation, voiced the suspicion that the reaction of inflation to external shocks is not stable over time but depends on the level or volatility of inflation itself.[1] Such changing dynamics would be particularly significant for central banking, impacting inflation forecasts and the expected outcomes of monetary policy actions. Specifically, inflation projections often hinge on assumptions regarding the speed and extent to which changes in producer prices are transmitted to consumer prices. These considerations are crucial when central banks aim to contain price pressures generated by supply shocks.[2] Relying on theory for this question is difficult, as alternative models of nominal rigidities, such as menu costs or Calvo pricing, yield different predictions for the pass-through of supply shocks to consumer prices. Consequently, identifying changing inflation dynamics also informs us about the validity of certain model assumptions.

We investigate this issue empirically by analyzing whether and when inflation dynamics undergo general changes. Using US data, we uncover two regimes by estimating a Markov-switching process based on inflation dynamics. Crucially, we do not tie the regimes to an exogenous indicator, such as an inflation threshold; rather, we allow the inflation process itself to determine them endogenously. It turns out that inflation volatility (quick changes in inflation rates) plays a more significant role in determining the regimes than its level. More precisely, regimes are predicted by inflation volatility: if annualized monthly inflation changes by more than 5.2 percentage points (pp.)—as in April, May, and July 2022—the economy is likely to switch to a high volatility regime.[3]

In a second step, we investigate state-dependent causal effects of a shock to the producer price index (PPI, provided by the the Bureau of Labor Statistics) on downstream price growth. Starting in 1948, we estimate how supply shocks to the crude materials PPI dynamically affect consumer prices. We also investigate the effects on intermediate stages of the production process, i.e., the PPIs for intermediate and finished goods. We rely on upstream PPI data as we are interested in more broadly defined supply shocks instead of price movements of a single input factor, which generalizes the results. Given that, e.g., crude materials display a much larger variance compared to consumer prices, PPI price processes are noisier. We, therefore, use movements in the crude materials PPI series that exceed normal fluctuations and move material prices and production in different directions as instruments for supply shocks.

[1]Philip Lane, Member of the Executive Board of the ECB, writes on November 25, 2022: "Our corporate contacts started [towards the end of 2021] expressing more concern about the persistence of input cost pressures, raising their price expectations for 2022 (also in view of rising energy prices). [...] Since the beginning of this year, many contacts also told us that prices would be increased more frequently." (Lane, 2022) More frequent price changes would alter the nature of the inflation process profoundly, as regards, e.g., the strength and speed of cost pass-through to inflation.

[2]See Sinn (2021) for an early warning of the 2021/22 surge in inflation based on rising producer prices and the implications for monetary policy.

[3]Here and the following, we use the words state and regime interchangeably.

Our results show that in periods of high inflation volatility, downstream prices, including the consumer price index (CPI), react much more strongly to cost shocks on impact and in subsequent months. Specifically, the maximum pass-through from the crude materials PPI to the CPI is 0.3% over a one-year horizon during the high-volatilty regime, while it is below 0.05% in the low-volatility regime.[4] In the high regime, prices are arguably more flexible and, hence, react more promptly to shocks. We validate our results for general supply shocks by estimating the responses to a specific one, i.e., oil-supply shocks as identified by Baumeister and Hamilton (2019). Again, the CPI exhibits a swifter and more pronounced reaction in the high-volatility state.[5]

Our 879-month sample allows us to disentangle episodes of large shocks, high inflation, or elevated inflation volatility. This separation is challenging because these phenomena are typically intertwined.[6] Our data, however, feature high-volatility episodes at both high and low inflation levels, and while some large shocks trigger high-volatility episodes, others do not. We hence recalculate regime-dependent CPI responses to input-price shocks, this time conditioning regimes exogenously on inflation volatility, the level of CPI inflation, or the size of the shocks. These exogenous splits strengthen state dependence with respect to inflation volatility, but do not yield comparable state dependencies for the inflation level or the size of shocks.[7]

Our findings suggest quicker price adjustments by firms in response to greater volatility in their sales markets. This explanation is supported by anecdotal evidence from the 2021/22 surge in inflation. Figure 1 depicts Google searches for the term 'Price escalation clause' alongside the change in the CPI inflation rate. If agreed upon in contracts between seller and buyer, these clauses automatically adjust sales prices based on changes in the seller's input costs.[8] That is, widespread use of these clauses implies a much faster price reaction to upstream cost changes, significantly altering inflation dynamics. Interest in this kind of clause is, as visible in the figure, correlated to the *change* in the inflation rate, peaking in the spring of 2021.[9] This coincided with a swift global rise in input prices due to several factors, among them strained global supply chains. Survey evidence corroborates this observation, as we find that 34% of sampled German firms in the Bundesbank Online

[4]We confirm that this difference is not driven by different reactions of monetary policy across regimes.

[5]Our findings square well with the observation in Borio et al. (2021) that 'salient,' i.e., large and positive, sectoral price movements displayed a lower pass-through to core PCE inflation during the great moderation, compared to previous periods. Smets et al. (2019) develop a multisector New–Keynesian model that accounts for 'pipeline pressures'.

[6]For instance, surges in inflation are rarely one-off; they often usher in prolonged periods of elevated and more variable inflation (see Blanco et al., 2025).

[7]When separating regimes based on the level of inflation, we use cutoffs at its average, the 65th, 70th, 80th, and 90th percentile, as well as at 5%. Apart from the impact period, we find no significant state dependence. We also replace regime splits with an interaction between supply shocks and inflation volatility, which again yields clear state dependence.

[8]The use of price escalation clauses is not just a recent phenomenon in the US; articles dating back to the 1940s already mention these clauses. For example, Mack (1946) describes different variations and provides advice for buyers facing escalation clauses.

[9]Figure C-1 in Appendix C compares this Google-search measure with the *level* of inflation, revealing a much less pronounced co-movement.

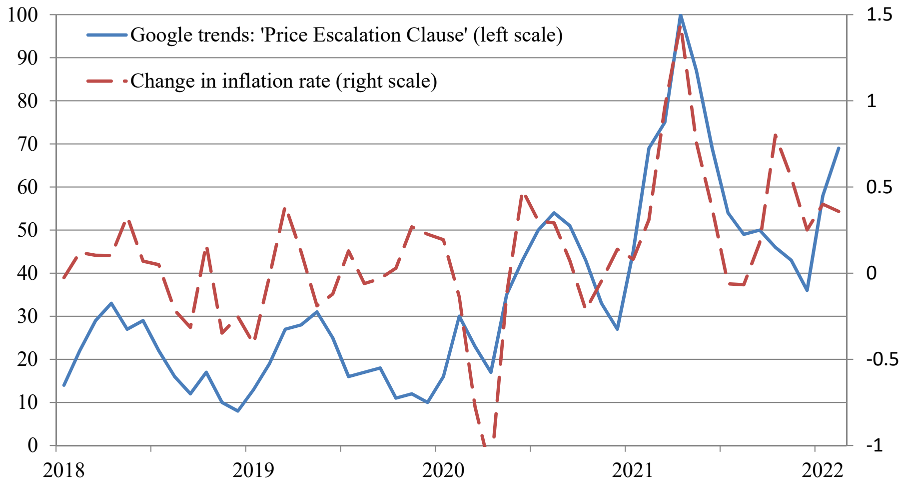

Figure 1: Escalation clauses. Index for Google searches of 'Price escalation clause' (left axis) and monthly change in annualized s.a. CPI inflation rate in percentage points (right scale).

Panel 2022 reported using price escalation clauses from 2021 onward, compared to only 17% before 2021.[10] This aligns with Figure 1, as inflation volatility peaked in 2021.

Regarding economic theory, our results, therefore, speak in favor of models in which prices react more quickly to shocks in the face of higher inflation volatility. We propose an analytically solvable model based on Devereux (2006) in which firms can invest in the flexibility of their prices.[11] The crucial difference to other models of state-dependent pricing, such as menu cost models, is the assumption that firms have an influence on price-setting costs if, in anticipation, they take adequate measures, such as using price-escalation clauses in new contracts.[12] In the presence of strategic complementarities in price setting, the payoff of being able to react quickly to new developments is higher in times of elevated inflation volatility. The heightened incentive to invest in price flexibility aligns with our finding of greater cost shock pass-through during periods of volatile inflation.[13] The model predicts a 'double dividend' to inflation targeting in terms of reducing inflation volatility, as it leads to a lower pass-through of shocks to inflation through the traditional direct

[10]For further information on the survey, see Deutsche Bundesbank (2025).

[11]We build our theoretical explanation on Devereux (2006) since his model setup captures the essential determinants for a firm's decision to invest in price flexibility in the most parsimonious way. Moreover, it represents a straightforward implementation of price-escalation clauses in a theoretical framework. Alternatively, but in a very similar spirit, observation costs in a menu cost model as in Álvarez et al. (2018) would also predict that higher volatility leads to more frequent price reviews and, hence, a higher cost pass-through. Rational-inattention models work in a related way (Mackowiak and Wiederholt, 2009). In menu cost models, as developed by, e.g., Golosov and Lucas (2007), the shape of the CPI response depends strongly on the shock size, which we do not find in our data for cost-push shocks. Standard Calvo price setting would not predict any state dependency at all.

[12]See Khalil and Lewis (2024) for a quantitative version of the model in Devereux (2006) that includes endogenous entry and exit of firms.

[13]Using micro data, Vavra (2014) shows that price changes become more dispersed during recessions, with a high dispersion when more products are changing prices. Our model aligns with these observations, as recessions typically feature higher volatility and higher volatility increases the share of firms that are able to change prices.

channel of altering demand, but also indirectly via reducing optimal price flexibility.[14] In contrast, monetary policy that is more accommodating in the face of supply shocks tends to increase price flexibility.

In a dynamic extension of the model, we follow Kimura and Kurozumi (2010) and allow firms to choose an optimal price-setting frequency, based on overall volatility and price-setting costs. The predicted inflation responses to supply shocks in high and low-volatility regimes are reasonably close to our empirical findings. In line with the analytical model, we find that stricter inflation targeting dampens the inflation response to cost-push shocks via an endogenous reduction of the price-setting frequency, on top of the standard demand channel. This effect is particularly strong for the high-volatility case.

Despite the important implications, surprisingly little research has focused on the pass-through of shocks to consumer prices in different inflation regimes, until recently. Given the policy relevance of this question, most research was conducted in policy institutions. By using Granger-Causality tests, Weinhagen (2002, 2016) demonstrates that upstream changes in prices explain price changes at each stage of production in the BLS PPI data, while more downstream price changes do not Granger-cause price changes. Bobeica et al. (2020, 2021) concentrate on the pass-through of labor costs to output prices, considering two regimes that depend on whether the level and volatility of inflation are above or below their historical means. Their findings, based on a Cholesky decomposition to identify labor cost shocks, indicate a quicker and more substantial pass-through in the high-inflation regime. De Santis and Tornese (2023) likewise document a stronger transmission of energy supply shocks on consumer prices in high-inflation regimes, while Ascari and Haber (2022) estimate more substantial price effects of monetary policy shocks—identified following Romer and Romer (2004)—during high inflation and for large shocks.[15] Similarly, the Bank for International Settlements (2022) investigates the pass-through of relative price changes, oil price shocks, and exchange rate movements into consumer prices, finding them to be dampened in periods of inflation below 5% (see also Borio et al., 2021, 2023).[16]

Our approach differs from the above studies in that we analyze the effects of general input-price shocks, derived by a novel identification scheme, on prices in later stages of production. Importantly, when identifying different inflation regimes, we do not impose a threshold of inflation or its volatility but let the inflation process itself determine the

[14] See also Kimura and Kurozumi (2010), as well as Paciello and Wiederholt (2014) for a related theoretical mechanism in a context of rational inattention.

[15] Berger and Vavra (2019) also find evidence for time-varying responsiveness of prices to shocks. Vavra (2014) favors a theoretical explanation based on a menu cost model featuring shocks to the volatility of idiosyncratic firm productivity, see also Hall (2023). Higher volatility reduces the effect of aggregate demand shocks on output in this context. As described above, our empirical results for supply shocks do not support pure menu cost models.

[16] Using household surveys, Bracha and Tang (2024) and Weber et al. (2025) show that consumers become more attentive in times of high inflation. Considering firms, however, Gautier et al. (2025) find that actual decisions became detached from inflation expectations during the latest inflation hike.

regimes. By doing so, we uncover the significance of inflation volatility in determining the regimes, a factor that has not been considered so far.[17]

We also contribute to the literature on the general pass-through of cost shocks.[18] A large part of this literature centers on the exchange-rate pass-through (see, e.g., Taylor, 2000; Campa and Goldberg, 2005; International Monetary Fund, 2006; Auer and Schoenle, 2016; Álvarez et al., 2017; Enders et al., 2018; Bonadio et al., 2019). A recurrent finding is a falling exchange-rate pass-through over time until recently, in line with our result that lower inflation volatility is associated with less frequent price adjustments. Amiti et al. (2019) and Muehlegger and Sweeney (2022) consider cost shocks more broadly and find strong strategic complementarities in price setting, an important element in our explanation of the role of CPI inflation volatility in price setting.[19]

The remainder of this paper is organized as follows. Section 2 outlines our methodology, including shock identification. Section 3 presents the results, with effects on intermediate stages of production in Section 3.3 and robustness checks discussed in Section 4. Section 5 develops the model, and Section 6 concludes.

2 Methodology

2.1 A Markov-switching model to detect inflation regimes

Using a Markov-switching autoregressive (MS-AR) model on log differences of US CPI data, we identify regimes characterized by distinct inflation dynamics. This type of model was introduced by Hamilton (1989). The basic modeling idea is that there are different states s_t of the AR model characterized by regime-specific model coefficients and error variances. A discrete first-order Markov process governs the transition between these states. In our setting, we restrict the model to have two states. The Markov process can

[17] In fact, the empirical literature on state-dependent inflation dynamics typically focuses on the effect of the level of inflation without separating it from the impact of its volatility (see, e.g., Álvarez et al., 2019). Note that we do not claim that inflation volatility is the only factor influencing price flexibility. However, we find it to be an important and so far under-researched one. We therefore focus on its effects in our empirical and theoretical analysis.

[18] Our paper is also related to studies on the price-setting behavior of firms. Given the vast number of significant contributions in this field, we cannot even give a partial overview of this literature here and thus focus on the most directly related studies.

[19] In a similar vein, Baslandze and Fuchs (2025) show that firms, including non-importers, increase prices in response to competitors' supply chain disruptions. Using surveys, Blinder et al. (1998) and Fabiani et al. (2005) find that firms hesitate to change prices due to the fear of losing customers to competitors. The importance of competitors' prices is further underlined by Dedola et al. (2022), who, employing micro data, ascertain that the pass-through of import cost shocks is lower for larger firms than for smaller ones, suggesting a role for strategic complementarities. Similarly, Gödl-Hanisch and Menkhoff (2023), also using micro data, show that the pass-through of individual cost shocks undershoots that of aggregate shocks by 40%, likely an effect of strategic complementarity. Moreover, they find a more pronounced pass-through for firms that are uncertain about their future business situation, aligning with our result of a higher pass-through in volatile times.

then be described by the following transition matrix:

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \quad \text{where} \quad p_{i,j} = Pr(s_{t+1} = j | s_t = i),$$

where inflation dynamics are allowed to differ across states 1 and 2:

$$\Delta CPI_t = \begin{cases} \nu_1 + A_{1,1}\Delta CPI_{t-1} + \cdots + A_{1,4}\Delta CPI_{t-4} + e_{1,t}, & \text{if} \quad s_t = 1 \\ \nu_2 + A_{2,1}\Delta CPI_{t-1} + \cdots + A_{2,4}\Delta CPI_{t-4} + e_{2,t}, & \text{if} \quad s_t = 2. \end{cases} \tag{1}$$

We explain ΔCPI_t (seasonally adjusted CPI data in monthly log differences) by an intercept ν_m and autoregressive terms of four lags, which all switch between $m = \{1, 2\}$ states, just like the variance of the residual term $e_{m,t}$.[20] We choose a rather small number of regimes and lags to keep the model as parsimonious as possible and thus to increase the reliability of the estimates. In this way, we also reduce computational cost significantly.

We estimate the model parameters and the hidden Markov chain with the expectation maximization (EM) algorithm.[21] We then obtain the filtered state probabilities $Pr(State_t)$, which we use for constructing the state indicator H_t (Chauvet and Hamilton, 2006). When the filtered probability of being in State 2 is greater than 0.5 in period t, H_t is assigned the value of 1, and 0 otherwise.[22] Correspondingly, the indicator for being in State 1 is $1 - H_t$.

2.2 State-dependent local projections

We follow the local projection instrumental variable (LP-IV) approach of Stock and Watson (2018) to construct the impulse responses. This method consists of a first-stage regression (2) in which the endogenous variable x_t is regressed on the instrument Z_t, and a second stage (3) that regresses the response variable y_t on the fitted values of the first stage, $\hat{x}_t$, and a set of (lagged) control variables W_t:

$$x_t = \mu_1 + \beta_1 Z_t + \sum_{l=1}^{n} \delta_{1,l} W_{t-l} + \epsilon_t \tag{2}$$

$$y_{t+h} = \mu_{2,h} + \beta_{LPIV,h} \hat{x}_t + \sum_{l=1}^{n} \delta_{2,l} W_{t-l} + u_{t+h}. \tag{3}$$

The coefficients $\hat{\beta}_{LPIV,h}$ then represent the impulse responses at each projection horizon h. $\hat{\mu}_1$ and $\hat{\mu}_2$ denote the intercepts, ϵ_t and u_t the error terms.

[20]Since we use monthly data, we also estimated an MS-AR including four lags plus the 12^{th} lag. We did not observe significant differences in the timing of the resulting regimes. The identified regimes are generally not sensitive to the lag length.

[21]For further explanation of the EM algorithm, see Hamilton (1990).

[22]Using smoothed instead of filtered probabilities does not change the results much. Our main results also remain unchanged if we assign periods to State 2 if the filtered probability is above 0.4 or 0.7, where in the latter case we have to reduce the number of lags in regressions (2) and (4) from 12 to 8, as we would otherwise end up with too few outliers in State 2, see below.

Adding to this core model, we interact the fitted values $\hat{x}_t$ and the controls W_t with a state indicator H_t taking the value 0 in State 1, and 1 in State 2. Modifying the second-stage equation (3) in this way allows us to estimate state-dependent impulse response functions (IRFs):

$$\begin{aligned} y_{t+h} =& \mu_{2,h} + (1 - H_t)(\beta^1_{LPIV,h}\hat{x}_t + \sum_{l=1}^{n} \delta^1_{2,l} W_{t-l}) \\ &+ H_t(\beta^2_{LPIV,h}\hat{x}_t + \sum_{l=1}^{n} \delta^2_{2,l} W_{t-l}) + u_{t+h}. \end{aligned} \tag{4}$$

The coefficients $\hat{\beta}^1_{LPIV,h}$ and $\hat{\beta}^2_{LPIV,h}$ form the impulse responses at horizon h in states 1 and 2 respectively. Estimation of equation (4) is done via ordinary least squares regression for each projection horizon h separately.

The sample we use to estimate our baseline model (4) for the United States is in monthly frequency and spans from October 1948 to December 2021. The endogenous variable x_t is the log difference of the crude materials producer price index (referred to as Crude PPI) of the Bureau of Labor Statistics' stage-of-processing (SOP) system. For the response y_t in the baseline model, we use log differences of the US CPI. In alternative setups, we also employ the SOP-PPI data for intermediate materials, supplies, and components (Intermediate PPI), and finished goods (Finished PPI) or the industrial production SOP data for crude goods (Crude IP) as dependent variables. Appendix A provides more details on the PPI and IP data.

2.3 Shock identification

To identify the causal effect of a producer price shock on consumer price inflation, we identify the effects of unexpected and unusual price movements, filtering out smaller ups and downs over time. Given the relatively high frequency of our data set (monthly), this approach makes us more confident that we identify actual shocks. To do so, we introduce a new identification approach and argue that outliers in time series data, which are often due to rare and unforeseen events, are correlated with the exogenous shocks that we wish to identify.[23] Specifically, we instrument producer prices with a variable based on data outliers in the respective PPI series and assume that outliers in the PPI series are correlated with structural producer price shocks but uncorrelated with other shocks. The outlier-based instrument, hence, satisfies the LP-IV relevance and contemporaneous exogeneity condition of Stock and Watson (2018).[24]

[23]Li et al. (2022) also follow a data-driven approach for shock identification as they identify shocks of Bitcoin and crude oil returns via the empirical quantiles of the two series. Kapetanios and Tzavalis (2010) show that well-known oil price shock events coincide with periods in which they find an outlier in their oil price data.

[24]Those are: i) Z_t must be relevant, i.e., the shock of interest $\eta_{j,t}$ must be correlated with the instrument: $E[\eta_{j,t}Z_t] \neq 0$, ii) Z_t must be contemporaneously exogenous to all other shocks $\eta_{-j,t}$: $E[\eta_{-j,t}Z_t] = 0$ and iii), Z_t must be exogenous to all shocks at all leads and lags: $E[\eta_{t+i}Z_t] = 0, \forall i \neq 0$.

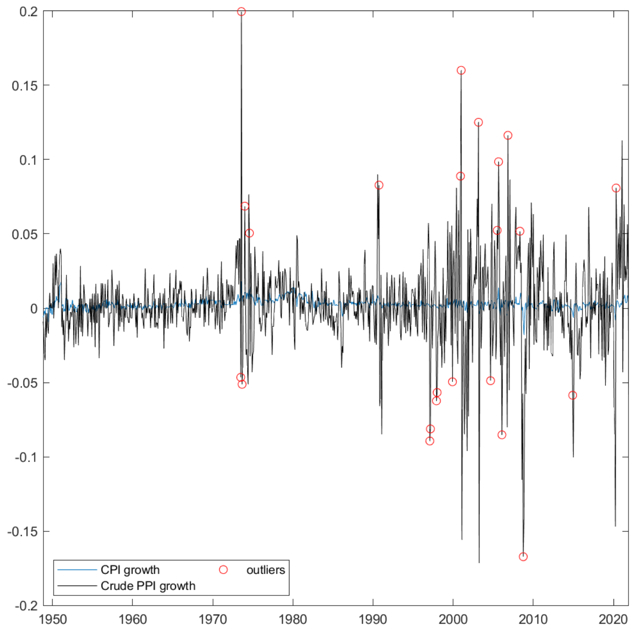

Figure 2: Crude PPI growth and outliers. Monthly growth rates (black) of the Crude PPI series, respectively, against monthly CPI growth (blue). Red circles mark the outliers generated with the iForest algorithm that survive the restriction described in equation (5).

To ensure that demand shocks are not the cause of the observed outliers, we only consider those outliers for which the materials industrial production index IP^M does not move contemporaneously in the same direction as the Crude PPI.[25] That is, we construct the outlier-based instrument Z_t in the following way:

$$Z_t = \begin{cases} 1, & outlier > 0 \quad \& \quad \Delta IP^M < 0 \\ -1, & outlier < 0 \quad \& \quad \Delta IP^M \geq 0 \\ 0, & else. \end{cases} \tag{5}$$

Z_t takes the value of 1 when there is a positive outlier in the PPI series and no positive movement in the IP series in period t. In case of a negative outlier and no negative change in the corresponding IP series, $Z_t = -1$, and $Z_t = 0$ if no anomaly is detected. To ensure that Z_t satisfies the third LP-IV condition (exogeneity to all shocks at all leads and lags), we follow Stock and Watson (2018) and include 12 lags of Z_t, y_t, $\Delta \log IP_t^M$, and the growth of the log of the Intermediate PPI, summarized in W_t, as controls in regressions (2) and (4). Furthermore, we include lags of Z_t as controls to correct for a possible correlation between the instrument and past values of the shock of interest. By including lags of the materials IP series as a monthly proxy for activity, we correct for any correlation between Z_t and earlier real developments. Controlling for lags of CPI and Intermediate PPI growth rules out the possibility that the instrument Z_t is correlated with a shock to consumer prices or the producer prices of the previous stage. This, in

[25]We use this IP index, provided by the Board of Governors, as it is available for our whole sample, starting in 1948.

addition to the restriction on ΔIP^M, further ensures that the dynamic effect we measure is not driven by a previous hike in demand leading to an increase in downstream prices first, followed by increasing upstream prices thereafter.

We detect outliers in the producer price indices using the isolation forest algorithm (iForest) proposed by Liu et al. (2012).[26] Instead of first defining normal instances in the data, the iForest directly detects anomalies through two quantitative properties: i) anomalies are the minority, and ii) they have attribute values different from those of normal instances. When setting the proportion of outliers in the PPI series (transformed to log differences) to 0.08, the algorithm detects 71 outliers.[27] Figure 2 shows the Crude PPI series and the detected outliers at which the materials IP index does not move in the same direction. The outliers coincide with prominent events on the supply side that led to large movements in Crude PPI inflation. Visible are the oil-price shock in 1973, which caused a spike in crude-material prices with a subsequent adjustment and re-escalation; the tensions surrounding the gulf war in 1990; the Asian crisis in 1997, which triggered a decline of Asian commodities demand (exogenous to the US) and an appreciation of the dollar; OPEC production cuts in early 2001; supply adjustments and a dollar appreciation following the financial crisis in 2008; and supply chain disruptions in 2021.

These shocks occur in phases of low and high inflation volatility. That is, a single outlier does not necessarily move the economy to a high-volatility regime, which might happen for a series of large and/or more frequent smaller shocks. For example, as shown below, turbulent oil prices in the 1970s induced switches to a high-volatility regime, while the Asian crisis did not.

3 Empirical results

We now turn to the results of the baseline specification. We first describe the identified regimes and then discuss the effects of producer price shocks within these regimes. Finally, we analyze the transmission on intermediate stages of production. Section 4 presents robustness checks. Specifically, we employ alternative regime definitions, which are independent of the Markov switching model and consider different state-dependencies, based on the level of inflation or the shock size. Alternatively, we rely on oil-supply shocks to measure supply shocks. We also examine whether the shock's sign affects the outcome, change the starting date, and analyze the interest-rate and exchange-rate responses. The conclusion remains the same: prevailing volatility significantly impacts the transmission of supply shocks.

[26] Specifically, we use the implementation in the Scikit-learn Python package by Pedregosa et al. (2011). For further explanations of the algorithm, see Liu et al. (2012).

[27] We choose 8% as lower values result in too few shocks and consequently weak instruments. Higher values might identify price movements that are not connected to clear supply shocks. We, therefore, prefer this rather conservative number. In any case, we also obtain state-dependent responses of inflation to shocks to Crude PPI for, e.g., half or double this value.

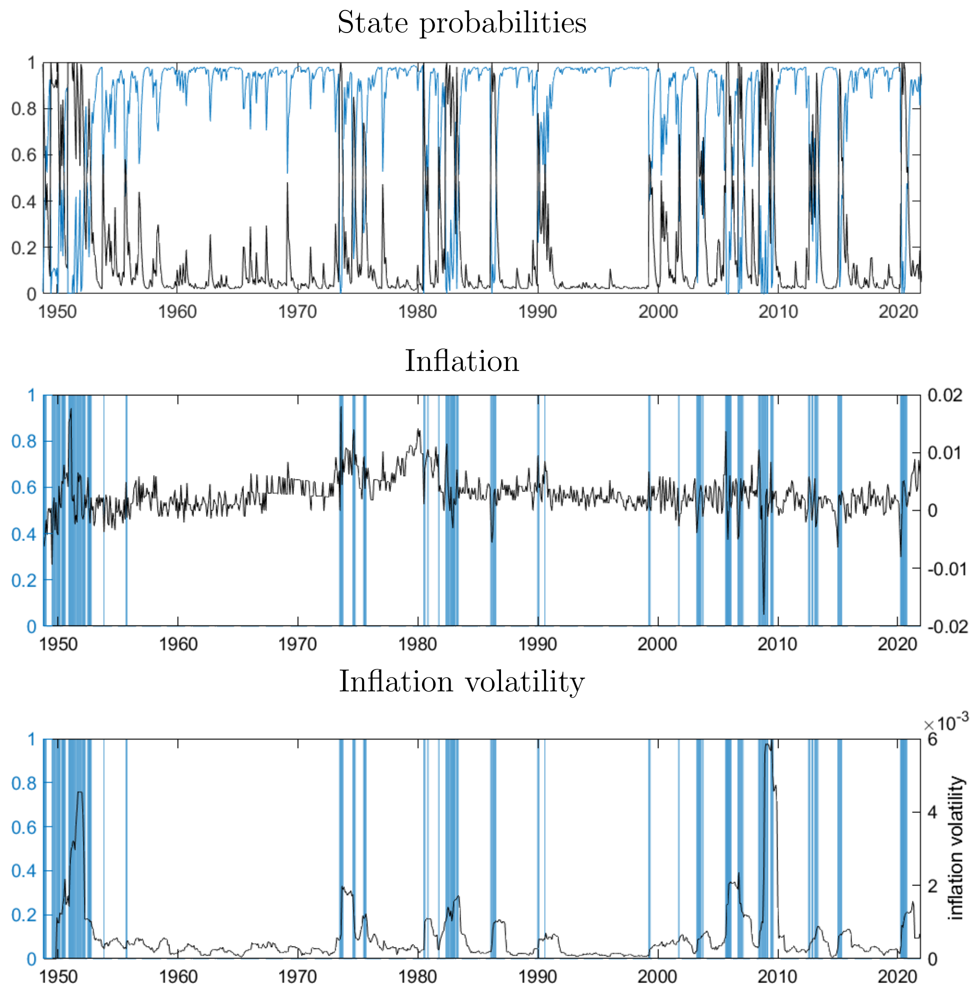

Figure 3: Identified regimes. Top panel: filtered state probabilities estimated from model (1); blue line: State 1, black line: State 2. Middle panel: monthly growth of CPI (black line); white areas: State 1, blue areas: State 2. Bottom panel: inflation volatility (black lines, variance of monthly CPI growth over a rolling window of 12 months); white areas: State 1, blue areas: State 2.

3.1 Identified regimes

Figure 3 shows the filtered state probabilities—estimated with the methodology described in Section 2.1—and the resulting state indicator H_t, in comparison with monthly growth rates of CPI and inflation volatility. We measure inflation volatility by the variance of monthly CPI growth over a rolling window of 12 months. As it turns out, the inflation regime is in State 2 when there are sudden swings in monthly CPI growth and generally increased volatility.[28]

[28] Corresponding to periods of high volatility, Gautier et al. (2024) find an increased frequency of price setting at the end of the 2000s in the period during and after the financial crisis, using European micro data from 11 countries over the period 2005-19. This evidence is in line with our theoretical interpretation of more flexible prices in State 2. Similarly, Dedola et al. (2023), making use of the same micro data, argue that recent evidence suggests that the return of higher and more volatile inflation seems to be associated with higher frequencies of price changes. Furthermore, Galeone and Gros (2023) find core inflation behavior to have shifted in the 2022/23 period as regards its magnitude, its rate of change, and its stickiness, as well as its responsiveness to energy prices.

Parameters	State 1	State 2
Std. dev. of monthly Δ CPI in %	0.27	0.56
Mean of monthly Δ CPI in %	0.34	0.24
Mean size outliers Crude PPI	0.06	0.07
CPI autocorrelation lag 1	0.75	0.48
CPI autocorrelation lag 2	0.65	0.21
Probability to stay in regime	0.97	0.87
Avg. state duration in months	33	7.7
Percentage of all outliers	70%	30%
Positive outliers	59%	57%

Variables	β	p-value	Variables	β	p-value
constant	-0.30	0.00	vol_{t-5}	0.02	0.36
vol_t	0.46	0.00	vol_{t-6}	0.02	0.37
vol_{t-1}	0.30	0.00	vol_{t-7}	0.05	0.04
vol_{t-2}	0.13	0.00	vol_{t-8}	0.03	0.14
vol_{t-3}	0.12	0.00	vol_{t-9}	0.03	0.24
vol_{t-4}	0.08	0.00	vol_{t-10}	0.00	0.94
R^2	0.69		*Adj.* R^2	0.68	
Obs.	589				

Table 1: Regime characteristics and determinants. Upper panel: characteristics of the two regimes. All statistics in percent. Lower panel: regression of filtered state probabilities on exogenous volatility indicator and its lags, maximizing R^2.

We report descriptive statistics for the inflation regimes in the upper panel of Table 1. Regimes differ primarily in the standard deviation of monthly inflation, with an average of 0.27 in State 1 and more than double (0.56) in State 2. The overall mean of monthly CPI growth is 0.34% in State 1 and only 0.24% in State 2, which highlights that not the level of inflation but rather its volatility characterizes the different inflation regimes. The higher volatility is only to a very low degree driven by larger outliers, as we find similar values for their mean values across regimes. Instead, the autocorrelation of monthly CPI growth seems to be more important for the differences. We calculate the autocorrelation up to two lags, considering only those regime realizations that consist of at least three consecutive periods. In State 1, we find a value of 0.75 for the first lag and 0.65 for the second, in contrast to 0.48 and 0.21 for lag one and two in State 2. The states are relatively persistent: The probability of staying in State 1 when being in the same state (i.e., p_{11}) is 0.97, and 0.87 for State 2 (p_{22}). This translates to an average state duration of 33 periods for State 1 and 7.7 periods for State 2. The states feature a similar percentage of positive relative to negative outliers, where 70% of all outliers occur in State 1.

The correlation between the state indicator and a volatility dummy variable vol_t—which equals 1 if the absolute change in the CPI exceeds its average, and 0 otherwise—is 30% and statistically significant. In addition, we regress the Markov-filtered state probabilities

$Pr(State_t)$ on vol_t as follows:

$$Pr(State_t) = c + \sum_{i=0}^{t=10} vol_{t-i}. \tag{6}$$

The contemporaneous indicator and the first four lags are significant at the 5% level.[29] Alternatively, we define the volatility indicator variable *vol* such that the R^2 of the mentioned regression is maximized, reaching 0.69, and find a threshold for the absolute value of the monthly change in CPI growth of 0.43 pp., or 5.28 pp. in annualized terms. That is, the optimized indicator variable takes the value of 1 if the absolute change in monthly inflation is above this threshold and zero otherwise. This value corresponds to approximately the 90th percentile of our sample; it was reached in, e.g., April 2022 (change in monthly inflation: -0.6 pp.), May 2022 (0.5 pp.), and July 2022 (-1.22 pp.). The correlation between the Markov state probabilities and this indicator is 0.65 and significant. The lower panel of Table 1 reports the resulting coefficients from regression (6) with the optimized threshold. If the current monthly absolute change in CPI growth is above 0.43 pp., the likelihood of being in State 2 increases by 46 pp. (significant at the 1% level), ceteris paribus. The first four lags are also significant at the 1% level with decreasing coefficients.

Results are very similar if we include the contemporaneous values of the monthly VIX index, the growth rate of industrial production, and trend inflation (obtained by HP-filtering monthly inflation rates): the contemporaneous value and the first four lags of volatility remain significant at the 1% level, while the adjusted R^2 increases to 0.71. The optimal threshold for the indicator is still 0.43 pp. of the change in CPI growth and the correlation of the Markov state probabilities with the indicator remains at 0.65. To sum up, if annualized monthly inflation changes by more than 5.2 pp., the inflation regime is likely to switch to State 2. Furthermore, the longer inflation is volatile, the higher the likelihood of reaching State 2.

3.2 Effects of supply shocks in different volatility regimes

The left panel of Figure 4 shows the responses of CPI to a unit shock to Crude PPI over a horizon of 12 months. We estimate regression (4) by setting y_t equal to the change in the CPI and report the cumulated responses. They are significantly different in states 1 and 2 over almost the whole horizon considered. Specifically, in State 2—the one associated with higher volatility in monthly CPI growth—CPI reacts faster and stronger. That is, we find clear evidence for state dependency of the CPI response to supply shocks, as the transmission of producer price shocks to consumer prices is stronger and quicker during a high-inflation-volatility regime than during times of more tranquil inflation.[30]

[29] A higher-than-average absolute change in the CPI increases the likelihood to be in State 2 by 17 pp. If the last four monthly absolute changes were also above average, the likelihood is 46 pp. higher.

[30] Inflation remains mostly higher in State 2 up to a horizon of 23 months and falls thereafter.

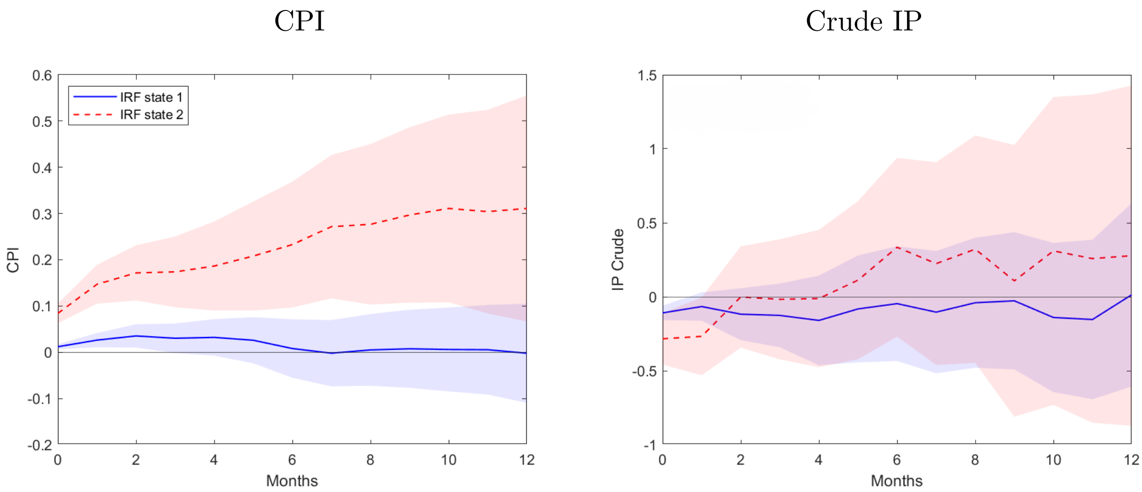

Figure 4: Baseline results. Impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of CPI to a shock to Crude PPI (left) and corresponding industrial production response (right). Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

A 1% surprise increase in Crude PPI raises the CPI by about 0.1 % on impact and up to 0.3 % over 12 months in the high-volatility regime, while the response in the low-volatility regime never exceeds 0.05%. The shaded areas represent 68% confidence bands. We construct them with Eicker-Huber-White (EHW) heteroskedasticity-robust standard errors as suggested by Montiel Olea and Plagborg-Møller (2021).[31]

In Appendix B we use the test of Lewis and Mertens (2022) to show that none of our instruments is weak, see the left panel of Figure B-1.Furthermore, the left panel of Figure C-3 in Appendix C demonstrates that the different CPI responses in the two regimes are not due to an endogenous reaction of monetary policy, i.e., a more expansionary monetary policy reaction in State 2.[32] Lastly, the right panel of Figure C-3 shows that the exchange rate appreciates more in the high-volatility regime, such that regime differences are not due to an exchange-rate depreciation that raises PPIs and the CPI alike.

We also calculate the effect of a shock to Crude PPI on industrial production of crude goods.[33] The right panel of Figure 4 depicts the results. As discussed in Section 2.3, to identify supply shocks we restrict industrial production to decrease in the period of a contractionary PPI shock. In the high-volatility regime, this effect is somewhat more pronounced, but the difference between regimes is much smaller compared to the CPI response and statistically not different from each other throughout.

[31] They show that when augmenting the local projection with lags of the response variable, EHW standard errors produce favorable results without the need to further correct for serial correlation in the regression residuals. In line with this argument, we include 12 lags of y_t in the local projection regressions.

[32] Specifically, monetary policy reacts more strongly to a shock to Crude PPI in State 2, in line with the larger CPI response.

[33] Given the availability of the Crude IP series, we move the starting date to January 1967. We also include its lagged values as controls.

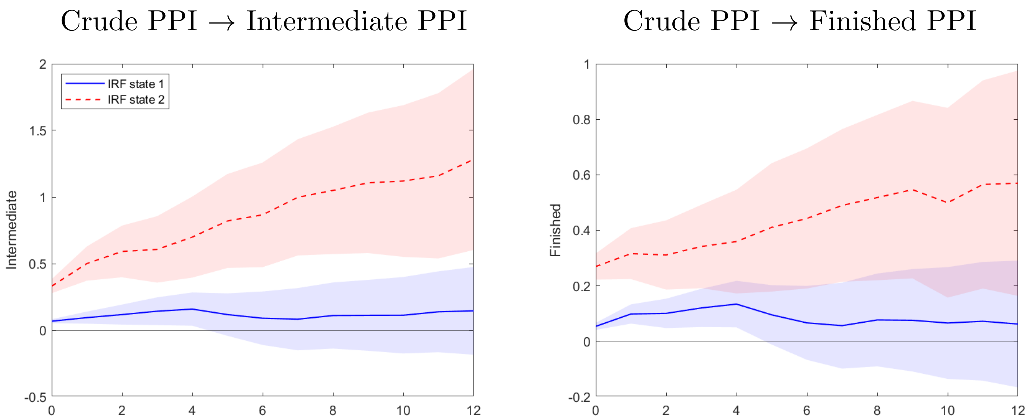

Figure 5: Effects of shocks to Crude PPI on intermediate stages. Impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of Intermediate PPI (left panel) and Finished PPI (right panel) to a shock to Crude PPI. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

3.3 Effects on intermediate stages of processing

Next, we analyze the effect of a shock to Crude PPI on the prices of products located downstream in the stages of processing system by setting the response variable y_t in (4) equal to Intermediate (left panel of Figure 5) or Finished PPI (right panel). Note that neither PPI includes imports. We add the corresponding industrial production data in the controls, which moves, due to data availability, the starting date to 1972M1.[34] We leave the rest of Model (4) unchanged. We again see significant differences between both states 1. These are quantitatively larger for Intermediate PPI than for Finished PPI, for which, in turn, the effect is larger than for the CPI. This is as expected, since at each stage of processing further inputs, such as labor, are added to the input materials.

4 Robustness

4.1 Alternative regime definitions

In this section, we depart from our regime identification via the Markov-switching model of Section 2.1 and employ alternative ways to define two separate regimes based on inflation volatility or alternative variables.

4.1.1 Inflation volatility

First, we define the high-volatility regime to prevail whenever inflation volatility, i.e., the change in CPI inflation, is above its sample average. That is, we use inflation volatility to exogenously separate regimes (rather than endogenously, as in our baseline). The left panel of Figure 6 presents the resulting impulse responses. State 1 (blue solid lines) covers

[34] We equate the industrial production index for primary & semifinished processing with Intermediate PPI and that of finished processing with Finished PPI.

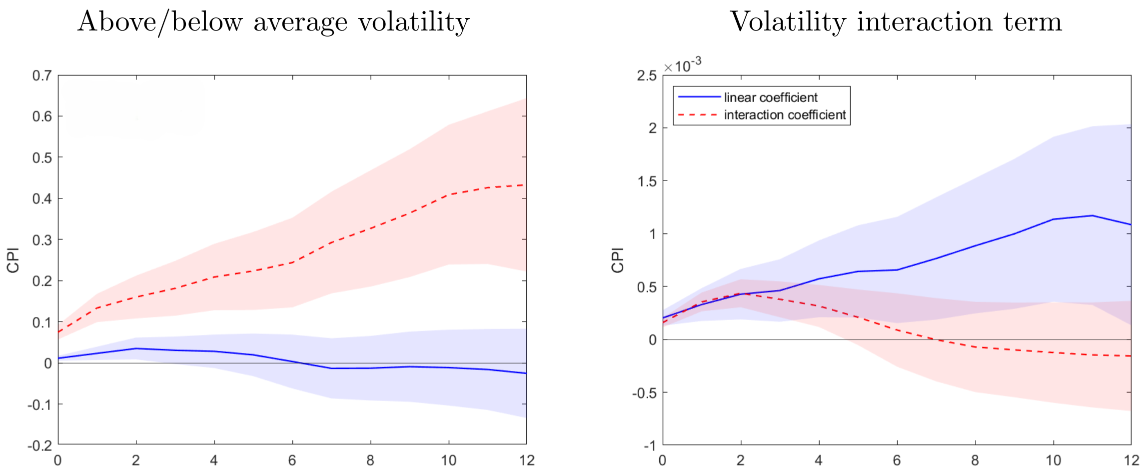

Figure 6: Alternative regime definitions. Left: impulse responses in Regime 1 (below-average volatility, solid blue line) and Regime 2 (above-average volatility, dashed red line) of CPI to a shock to Crude PPI. Right: linear (solid blue line) and volatility-interacted (dashed red line) effect of a shock to Crude PPI on CPI. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

below-average inflation volatility, calculated as in Figure 3, while State 2 (red dashed lines) captures above-average volatility. Consistent with—and even more pronounced than—our baseline in Figure 4, supply shocks to Crude PPI transmit to consumer prices both more rapidly and more strongly in the high-volatility regime.

Next, we abandon the discrete regime split and, instead, add the term $v_t \cdot \hat{x}_t$ in Model (3), where v_t represents inflation volatility. That is, we measure the effect of supply shocks depending on the prevailing level of volatility, in addition to its linear impact. Note that this specification is more restrictive than our baseline, as it assumes that volatility scales the impact of supply shocks in a linear way ——unlike our baseline, which allows for a discrete change. In the right panel of Figure 6, the blue solid line shows the linear effect of supply shocks on the CPI, while the red dashed line plots the additional effect from the interaction. The interaction term remains significantly positive over a prolonged horizon, confirming that supply shocks hit consumer prices more strongly when volatility is high.[35]

4.1.2 Inflation level and shock size

As stated in Section 2.1, the Markov-switching model indicates that regimes are separated by their inflation volatility rather than the level of inflation itself. In the following, we investigate whether alternative regime definitions based on the inflation level or the shock size result in similar state-dependent transmission of supply shocks. To this end, rather than relying on the regimes estimated by the Markov-switching model, we redefine regimes such that State 1 occurs when inflation is below its mean and State 2 when it is above. The left panel of Figure 7 shows the results for regimes below (blue solid lines) and above

[35]We standardized the volatility measure, such that low volatility levels attenuate the effect of the linear term, whereas values above the mean amplify it.

Inflation level above/below:

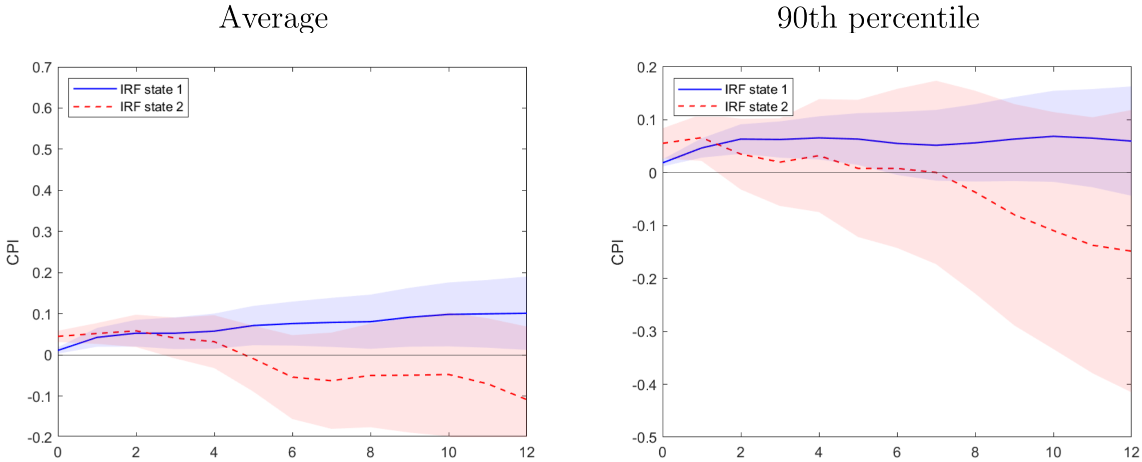

Figure 7: Regimes based on inflation level. Impulse responses in Regime 1 (low state, solid blue lines) and Regime 2 (high state, dashed red lines) of CPI to a shock to Crude PPI. State 1/2 if level CPI inflation is below/above average (left) or its 90th percentile (right). Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

(red dashed lines) the average inflation level. Only a small state dependency is visible. In particular, while the impact response of CPI inflation after a shock to Crude PPI is slightly higher than in State 2, it is below State 1 in the following periods.

To confirm robustness, we replicate the analysis using alternative inflation-level thresholds to separate the regimes. The right panel of Figure 7 applies a 90th-percentile cutoff, while the left panel of Figure C-2 in Appendix C uses the 70th percentile. In both cases, we observe no persistent state dependency beyond the impact period.[36] We also use an inflation level of 5% as a cutoff value.[37] The right panel of Figure C-2 presents the result. As before, the high-inflation state shows a slightly larger initial impact of a supply shock on inflation, but this significant difference dissipates after one month.

Next, we turn to the effects of the shock size. In standard menu cost models without observation costs (such as Golosov and Lucas 2007), price-setting behavior depends on the size of contemporaneous shocks. A central result is that large input-price shocks have a relatively larger impact on consumer prices compared to smaller ones, see Ascari and Haber (2022). Given that periods of higher inflation volatility could be correlated to the average shock size in these periods, we check whether this correlation can explain the above findings. Figure 8 shows the reaction to small versus large shocks. We pursue two alternative strategies. In the left panel, we again follow the approach of Ascari and Haber (2022) and add the term $|\hat{x}_t| \cdot \hat{x}_t$ in Model (3). We hence measure the effect of the squared shock but conserve the sign of the shock. We do this independently of the regimes, as we are here interested in the effect of the shock size as an alternative explanation for

[36]Results for the 65th and 80th percentiles are similar and available upon request.

[37]The Bank for International Settlements (2022) find that the CPI's response to shocks is attenuated when inflation is below 5% across a broad set of countries, while Álvarez et al. (2019) show that Argentine firms increase the frequency of price adjustments once inflation exceeds 5%.

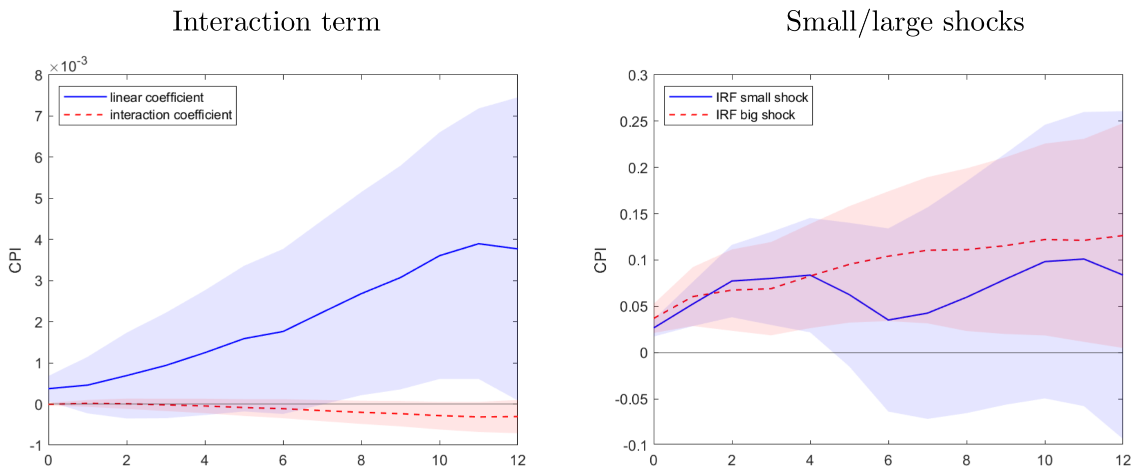

Figure 8: Effects of large vs. small shocks. Impulse responses of CPI to a shock to Crude PPI. Left: specification including linear (solid blue lines) and interaction term $|\hat{x}_t| \cdot \hat{x}_t$ (red dashed lines). Right: shock sizes below (solid blue lines) or above (dashed red lines) average. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

our results. The effects of input-price shocks on the CPI via this interaction term and the linear coefficient are plotted by red dashed and blue solid lines, respectively. The interaction term is either insignificant or even negative, showing that large supply shocks do not automatically lead to a larger pass-through compared to smaller shocks.[38] If, however, several larger shocks (or a series of smaller shocks) result in higher CPI volatility, the shock transmission is profoundly altered, see above.

In the right panel of Figure 8, we separate the outliers, as identified in Section 2.3, depending on whether they are larger or smaller than the average. As in the previous exercise, we do not find a significant difference between the effects of large vs. small shocks. That is, the influence of inflation volatility on the effect of supply shocks cannot be explained by the differential effects of the shock size.

4.2 Alternative shocks: oil supply shocks

We now turn to an alternative scheme for identifying supply shocks. Specifically, we exchange our identified shocks with oil supply shocks, i.e., a series of supply shocks that are well established in the literature. We use the oil supply shocks from Baumeister and Hamilton (2019), which range from February 1975 to December 2022. We again investigate possible differences in the CPI response in the two regimes identified in Section 3.1. The left panel of Figure 9 shows the results. As is visible, the effects are similar to our broader-based supply shocks of the baseline specification. Specifically, the effects of a supply shock are stronger on impact and thereafter in the high-volatility State 2.

[38] Given that Ascari and Haber (2022) consider the effects of monetary policy shocks instead of supply shocks, our results do not contradict their findings. For example, the effects of monetary policy decisions depend to a large degree on central bank communication and media coverage, influencing expectations, which might work quite differently depending on the size of the shock.

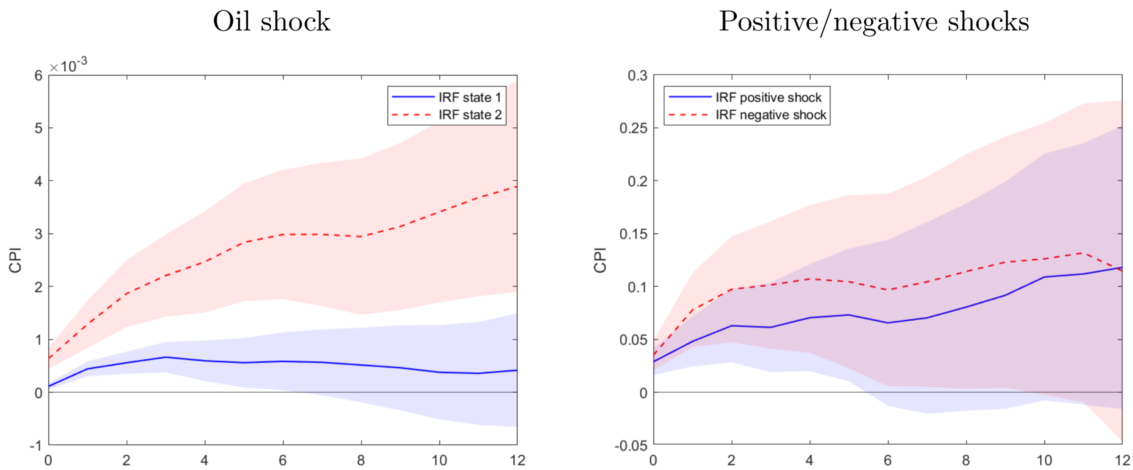

Figure 9: Effects of oil supply shocks and sign of shocks. Left: impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of CPI to a contractionary oil price shock by Baumeister and Hamilton (2019). Right: impulse responses to positive (solid blue lines) and negative (dashed red lines, positive values) of CPI to a shock to Crude PPI. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

4.3 Further robustness

In this section we explore the robustness of our results with regard to the sign of the shock, different samples, regression setups, and identification schemes.

First, we analyze potential asymmetries between positive and negative shocks. We first create an instrument containing only the positive outliers and then a second one with only negative outliers. We estimate both directions of the shock at the same time to avoid potential biases by truncated variables (Garzon and Hierro, 2021):

$$y_{t+h} = \mu + \beta_h^+ \hat{x}_t^+ + \beta_h^- \hat{x}_t^- + \sum_{l=1}^{n} \delta_{2S,l,1}^T W_{t-l} + u_{t+h}. \tag{7}$$

In Model (7), $\hat{\beta}_h^+$ and $\hat{\beta}_h^-$ denote the positive and negative impulse responses, respectively. $\hat{x}_t^+$ and $\hat{x}_t^-$ are the fitted values from a regression of the dependent variable x_t (Crude PPI) on the positive or negative instrument and lagged controls W_t, which are the same as employed in Model (4). The right panel of Figure 9 reports the resulting CPI responses to positive (solid blue lines) or negative (red dashed lines, positive values for ease of comparison) shocks to Crude PPI. The point estimates are fairly similar and confidence intervals overlap at all horizons. That is, the direction of the shock hardly changes the shape of the responses. An uneven distribution of positive versus negative shocks is, therefore, not responsible for the documented state dependency.

Second, we change the lag length to eight (instead of 12) in regressions (2) and (4) and show the results in the left panel of Figure 10. Again, results do not change much.

Third, we explore the possibility that the identified regimes depend on the dependent variable. Specifically, as shown by Gonçalves et al. (2024), if a shock affects the response variable y_t, it could also alter the state indicator H_t, if this depends on y_t. This might

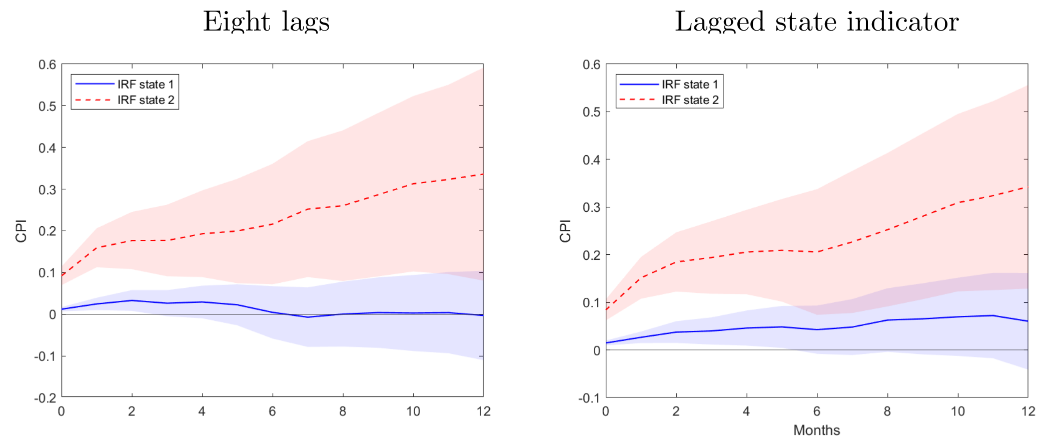

Figure 10: Robustness I. Impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of CPI to a shock to Crude PPI. Left: eight lags. Right: including state indicator as control. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

affect the state-dependent LP estimands and thus generate a bias in the impulse response. In our baseline, we nonetheless assume that a one-time unit shock will not induce an alternation of the states, as the regimes feature a high persistence of 33 months in State 1 and almost 8 months in State 2. Additionally, we follow Ramey and Zubairy (2018) and Cloyne et al. (2023) by lagging the indicator variable in regression (4). Results remain similar to our baseline, see the right panel of Figure 10. We also regress the state indicator variable on the contemporaneous and three lags of the fitted values of equation (2). None of the coefficients is significant.

Fourth, we move the sample start to 1969, as in Ascari and Haber (2022), in the left panel of Figure 11. In the right panel, we use 1972 as the starting date, after the peg of the dollar to gold was cut and towards the end of regulated oil prices in the US. Both versions show only minor differences and closely match our baseline estimates.

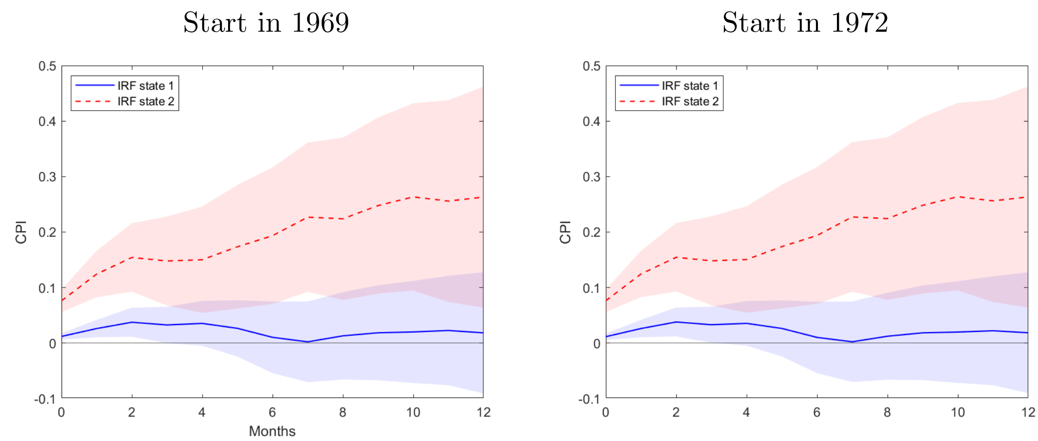

Figure 11: Robustness II. Impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of CPI to a shock to Crude PPI. Left: start in 1969M1. Right: start in 1972M1. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

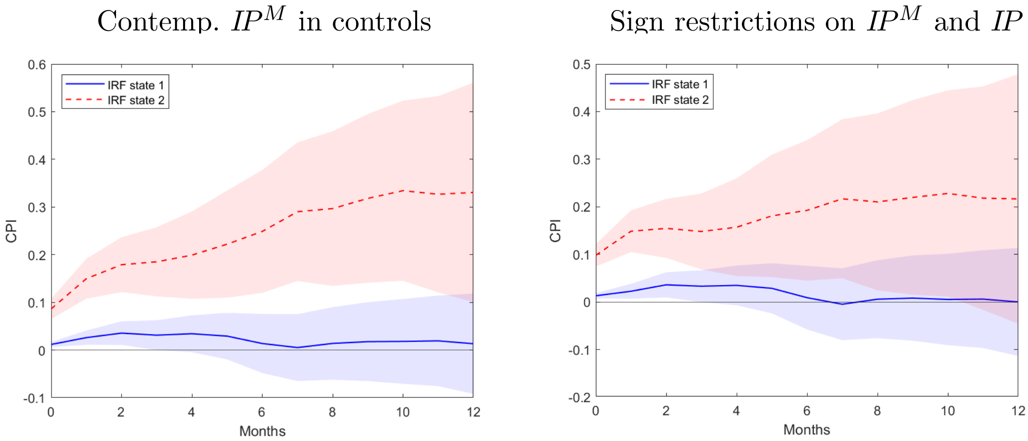

Figure 12: Robustness III. Impulse responses in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) of CPI to a shock to Crude PPI. Left: contemporaneous value of IP^M included in controls. Right: sign restrictions on IP^M and overall IP employed. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

Lastly, we test different specifications of the local projections to further demonstrate that we do not pick up demand shocks in our analysis. In particular, we include the contemporaneous value of IP^M (in addition to its lags) in the regression. The left panel of Figure 12 displays the results, which are similar to the baseline. The right panel of Figure 12 depicts the case in which we do not only restrict IP^M to have the opposite sign as Crude PPI, but both IP^M and overall industrial production.[39] Again, results change only mildly.

5 Implications for theory

We now turn to potential theoretical explanations for our empirical finding of a stronger and quicker transmission of input prices to consumer prices in times of high inflation volatility. Our preferred theory assumes that firms are able to invest in price flexibility. In Section 5.1, we first rely on the mechanism developed by Devereux (2006) in a one-period model to derive analytical results. Observation costs as in Álvarez et al. (2018) or models of rational inattention (Mackowiak and Wiederholt, 2009) could also account for our evidence. The core intuition underlying these models is the same: firms adjust their future price-setting behavior in response to current observations, where costs arise either from acquiring information or from changing prices. Yet, Devereux's mechanism is much simpler while leading to very similar conclusions. It can also be seen as a direct implementation of price escalation clauses into a standard pricing model. Our version is deliberately kept simple, since we aim to derive analytical results and to develop an intuition that could be used in several larger models. We then move on to derive quantitative predictions in an infinite-period version, relying on the mechanism proposed by

[39]We also include overall IP in the controls in this specification.

Kimura and Kurozumi (2010) and others. Here, firms invest in price flexibility by choosing the probability of being able to set prices in future periods, where higher flexibility is associated with larger costs.

We discard explanations based on menu costs or Calvo pricing with a fixed Calvo parameter for the following reasons. In standard menu cost models without observation costs (such as Golosov and Lucas 2007), price-setting behavior depends on the size of contemporaneous shocks. A central result is that large input-price shocks have a larger impact on consumer prices than smaller ones. This prediction can be tested in our data, see Figure 8 for the reaction to large versus small shocks. As discussed in Section 4.1.2, we do not find a significant difference between the effects of large and small shocks.

Calvo pricing with a fixed Calvo parameter, on the other hand, would predict a constant impact of cost changes on inflation and is, therefore, clearly unable to replicate a state-dependent pass-through.

5.1 Analytical model

We now sketch our preferred theory in a one-period model. We deviate from the original model in Devereux (2006) by introducing raw input material and a reaction function for the central bank—the model then features demand, supply, and monetary policy shocks—as well as simplifying the model by reducing it to a closed-economy setup and assuming pre-set wages. The following description of the model setup largely follows Devereux (2006), where more detailed derivations can be found. We introduce more significant changes to the original model in Section 5.1.2 and list the corresponding calculations in Appendix D. Model predictions are derived in Section 5.1.3.

5.1.1 Setup

Households maximize a utility function

$$U_t = \sum_{t=0}^{\infty} \log C_t - \frac{L_t^{1+\zeta}}{1+\zeta},$$

subject to the budget constraint $C_t P_t + B_t = (1+i_t)B_{t-1} + W_t L_t + C_{R,t} R_t + \Pi_t$, with $L_t = \int_0^1 L_{j,t} dj$; C_t is consumption, $L_{j,t}$ is hours worked at firm j, R_t is the aggregate input of raw materials, sold by households to firms, $C_{R,t}$ is their price, Π_t are profits or losses (including price setting costs) from firms, and B_t are nominal bonds that pay $1+i_t$ in period $t+1$.[40] W_t is the wage, which is equal for all firms. Consumption bundles are composed of infinitely many varieties of goods:

$$C_t = \left(\int_0^1 C_{j,t}^{(\varepsilon-1)/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)},$$

[40]This setup is equivalent to a structure in which raw materials producing firms are owned by households. As we model a closed economy, we assume that raw materials are available with unlimited supply at a fixed price $C_{R,t} = C_{RR,t} W_t$, where the relative price $C_{RR,t}$ of raw material to labor is exogenously given.

where $\varepsilon > 1$ is the elasticity of substitution between differentiated goods and market clearing implies $Y_{j,t} = C_{j,t} \forall j, t$. The aggregate price index is then

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.$$

This setup gives rise to a standard demand function

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y,$$

with $P_{j,t}$ as the output price of firm j. Y_t represents total demand in the economy. As we will consider only one period in this version of the model, we drop time indexes in the remainder of this section.

Now consider firm j that produces according to

$$Y_j = (I_j - D_j \Phi(j))^{\alpha}, \tag{8}$$

where $I_j = R_j^{\gamma} L_j^{1-\gamma}$ represents firm j's usage of a combined input factor consisting of raw material R_j and employment L_j.[41] $\Phi(j)$ is a firm-specific cost of price flexibility. The parameter $0 < \alpha < 1$ measures the degree of decreasing returns to scale. The indicator variable D_j equals one if the firm chooses to have ex-post flexible prices in the period under consideration and zero if it decides to forego the opportunity of setting prices after observing this period's shock realizations. In our context, we interpret this cost as, e.g., using price-escalation clauses, which might require price discounts to clients and/or additional legal advice. Similarly, preserving price flexibility by using contracts that cover only short periods instead of fixing prices for longer may cause costs, such as lower negotiable output prices and more frequent contracting costs.

A related, but more complex, mechanism relies on 'observation costs,' proposed by Álvarez et al. (2018). In our model, $\Phi(j)$ would then be a shortcut to costs arising from a closer market observation. These costs would induce firms to monitor economic developments more thoroughly in times of higher volatility, while the model of Devereux (2006) relies on higher investments in price flexibility. Both models predict that current observed volatility raises the responsiveness of prices to future shocks, which will be crucial for accounting for our findings. That is, even large supply shocks transmit to consumer prices only to a low degree if they happen in tranquil times. This prediction differentiates these models from other approaches, such as menu cost models without observation costs, discussed above.

The price MC for one unit of the input factor I consists of the wage W, which is set in advance and is therefore fixed in this one-period model, and of the price of the raw material C_R. The latter is stochastic, and so are Y and P, as seen from the firm's perspective. As usual, minimized costs for one unit of I are then

$$MC = \frac{C_R^{\gamma} W^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}. \tag{9}$$

[41] We fix capital at unity, as we are mainly interested in the short-term decisions of firms.

We refer to unexpected movements in the costs of raw materials as supply shocks. Expected discounted profits of the firm are

$$E\Gamma\left[P_j\left(\frac{P_j}{P}\right)^{-\varepsilon}Y - MC\left(\left(\frac{P_j}{P}\right)Y\right)^{\frac{1}{\alpha}} - MC D_j \Phi(j)\right],$$

where E is the expectational operator and $\Gamma = 1/(PY)$ is the stochastic discount factor of the firm, corresponding to the marginal utility of one dollar of a hypothetical household with log utility. If the firm chooses to pay the (known, idiosyncratic) costs $\Phi(j)$, it can adjust its price after observing MC, Y, and P; otherwise, it sets its price based on expectations regarding these variables. The optimal price for firms that have chosen to invest in price flexibility is

$$P_j^1 = \delta\left[MC^{\alpha}(\hat{Y})^{1-\alpha}\right]^{\omega}, \tag{10}$$

where $\delta = (\varepsilon/[\alpha(\varepsilon-1)])^{\alpha\omega}$ and $\omega = 1/[\alpha+\varepsilon(1-\alpha)]$. Furthermore, $\hat{Y} = P^{\varepsilon}Y$ is the part of a firm's demand that is independent of its price. Firms that choose to set their price in advance do this according to

$$P_j^0 = \delta\frac{E\left[\Gamma MC(\hat{Y})^{\frac{1}{\alpha}}\right]^{\alpha\omega}}{E\left[\Gamma\hat{Y}\right]^{\alpha\omega}}. \tag{11}$$

Expected profits under optimal price setting then depend on the choice to invest in price flexibility in the following way

$$\begin{aligned} V^1(\Theta) &= \Psi E\Gamma(MC^{\alpha(1-\varepsilon)}\hat{Y})^{\omega} \\ V^0(\Theta) &= \Psi(E\Gamma MC\hat{Y}^{1/\alpha})^{(1-\varepsilon)\alpha\omega}(E\Gamma\hat{Y})^{\varepsilon\omega}, \end{aligned}$$

where $V^1(\Theta)$ are profits for $D_j = 1$ and $V^0(\Theta)$ for $D_j = 0$. The parameter Ψ equals $\delta^{1-\varepsilon} - \delta^{-(\varepsilon/\alpha)}$ and $\Theta = \{C, Y, P\}$. The firm chooses ex-post price flexibility whenever the difference in expected profits for $D_j = 1$ and $D_j = 0$ is higher than the discounted costs of investing in price flexibility, i.e., if $V^1(\Theta) - V^0(\Theta) \geq \Phi(j) E\,\Gamma\, MC$, or

$$\Delta(\Theta) = \frac{V^1(\Theta) - V^0(\Theta)}{E\,\Gamma\, MC} \geq \Phi(j). \tag{12}$$

$\Delta(\Theta)$ is the discounted gain from investing in price flexibility, normalized by the cost of the combined input factor. This equation can be solved by taking a second-order approximation around the mean value $E\ln\Theta$, see Devereux (2006) for details:

$$\Delta(\Theta) \approx \frac{\Omega\alpha}{2}Var\left(\ln MC + \frac{1-\alpha}{\alpha}\ln\hat{Y}\right) = \frac{\Omega\alpha}{2}\left[\sigma_{mc}^2 + \left(\frac{1-\alpha}{\alpha}\right)^2\sigma_{\hat{y}}^2 + 2\frac{1-\alpha}{\alpha}\sigma_{mc,\hat{y}}\right] > 0, \tag{13}$$

where lower-case letters stand for percentage deviations from the stochastic steady state, such as $mc = \ln MC - E\ln MC$. Furthermore, $\Omega = [V(\exp(E\ln\Theta))/\exp(E(\ln\Gamma +$

$\ln MC))]\varepsilon(\varepsilon-1)\omega^2 > 0$, where $V(\exp(E\ln\Theta))$ are profits evaluated at the mean $E\ln\Theta$ and $\sigma^2_{mc}, \sigma^2_{\hat{y}}, \sigma_{mc,\hat{y}} > 0$ are the variances of input costs and market demand, as well as their covariance. Given expression (9), the cost variance σ^2_{mc} depends on the variance of (the log of) raw material costs in the following way: $\sigma^2_{mc} = \gamma^2\sigma^2_{c_R}$. Equations (12) and (13) deliver an important insight in line with our empirical findings: higher volatility $\sigma^2_{\hat{y}}$ of market demand $\hat{Y} = P^{\varepsilon}Y$, which itself depends on price volatility, increases the incentives for firms to invest in price flexibility. In Section 5.1.3, we will show that $\Delta(\Theta)$ is directly related to inflation volatility, such that inflation volatility determines price flexibility.

5.1.2 Closing the model

We now close the model, leading to several differences to Devereux (2006). Assume that there is a unit mass of firms. We then rank firms according to their cost of investing in price flexibility. The firm with the index $j = 0$ has the lowest costs $\Phi(0) = 0$ and the one with $j = 1$ the highest. We also assume that $\Phi(j)$ is uniformly distributed and differentiable. Denote the index of the firm that is indifferent to whether to invest in price flexibility or not as z. That is, z is the measure of firms that do invest. The resulting value of z is determined by the following conditions

$$\Delta(\Theta) = \Phi(z), \quad 0 \leq z < 1, \tag{14}$$
$$\Delta(\Theta) > \Phi(1), \quad z = 1. \tag{15}$$

The overall price index for a given value of z is then

$$P = \left[z(P^1)^{1-\varepsilon} + (1-z)(P^0)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

Nominal demand is determined by the money supply in the following way

$$YP = \frac{M}{\chi}, \tag{17}$$

where χ features i.i.d. shocks to velocity and has an expected value of unity.[42] We refer to these shocks as the demand shock from now on. Inserting equation (17) into the optimal prices of firms (10) and (11), while observing that all firms that can adjust set the same prices, results in

$$P^1 = \delta\left[MC^{\alpha}P^{(1-\alpha)(\varepsilon-1)}(M\nu/\chi)^{1-\alpha}\right]^{\omega} \tag{18}$$
$$P^0 = \delta\frac{E\left[\Gamma MC\left(P^{\varepsilon-1}(M\nu/\chi)^{1-\alpha}\right)^{\frac{1}{\alpha}}\right]^{\alpha\omega}}{E\left[P^{\varepsilon-1}\right]^{\alpha\omega}}. \tag{19}$$

The central bank sets the change in the nominal money supply based on current inflation:

$$\frac{M}{M_{-1}} = \left(\frac{P}{P_{-1}}\right)^{-\phi}\nu, \tag{20}$$

[42]These shocks can be derived from shocks to households' preference for holding money, see Devereux (2006).

where we normalize the previous period's values of the money stock and the price level to unity $M_{-1} = P_{-1} = 1$. We assume that the central bank does not react to higher inflation by increasing the money supply overproportionally, i.e., $\phi \geq -1$. Stricter inflation targeting corresponds to a higher value of ϕ. The variable ν with an expected value of unity may stand for monetary policy shocks, but also for systematic deviations from a rule that focuses on inflation only. In particular, we allow for a positive correlation between ν and the supply shock, which represents a monetary policy strategy that is relatively more accommodating in case of supply shocks.[43] Theoretically, ν could also be linked to demand shocks. Given the debate in some policy circles surrounding lower reactions to inflation in case of supply shocks, we focus on a correlation with this kind of shock.[44]

To derive the expression for equation (13) in general equilibrium, we use the linearized price index (18) together with the linearized versions of equations (16) and (20), see Appendix D. This yields

$$p = \frac{\varphi(z)\omega}{\Delta}\left[\alpha mc + (1-\alpha)(\hat{\nu} - \hat{\chi})\right], \tag{21}$$

with

$$\Delta = 1 - \varphi(z)\omega(1-\alpha)(\varepsilon - \phi - 1),$$

where $\hat{\chi} = \ln\chi - E\ln\chi$ and $\hat{\nu} = \ln\nu - E\ln\nu$. The parameter $\varphi(z)$ is given in the appendix and follows $\varphi(0) = 0, \varphi(1) = 1, \varphi'(z) > 0, \varphi''(z) > 0$. Using equation (21) we derive—again in the appendix—the variance of $\ln MC + \frac{1-\alpha}{\alpha}\ln\hat{Y}$ and use this in equation (13) to arrive at equations (14) and (15) in general equilibrium as

$$\frac{\Omega\alpha}{2\Delta^2}\left[\sigma_{mc}^2 + \left(\frac{1-\alpha}{\alpha}\right)^2(\sigma_{\hat{\chi}}^2 + \sigma_{\hat{\nu}}^2) + 2\frac{1-\alpha}{\alpha}\sigma_{mc,\hat{\nu}}\right] = \Phi(z) \qquad 0 \leq z < 1 \tag{22}$$

$$\frac{\Omega\alpha}{2\Delta^2}\left[\sigma_{mc}^2 + \left(\frac{1-\alpha}{\alpha}\right)^2(\sigma_{\hat{\chi}}^2 + \sigma_{\hat{\nu}}^2) + 2\frac{1-\alpha}{\alpha}\sigma_{mc,\hat{\nu}}\right] > \Phi(1) \qquad z = 1, \tag{23}$$

The covariance $\sigma_{mc,\hat{\nu}}$ corresponds to $-\phi_{mc}\sigma_{mc}$, see footnote 43.

5.1.3 Model predictions

Equations (22) and (23) then determine the equilibrium value of z, depending on the variances and covariances of the three shocks. As shown in the appendix, there can be one or three equilibria. However, in case of multiple equilibria, one is unstable. In the following, we focus on the description of the stable equilibrium in which the economy is not already at complete price flexibility (i.e., $z < 1$).[45] We first assert the relation between

[43] The functional form would be $\nu = (MC/MC_{-1})^{-\phi_{mc}}\tilde{\nu}$, with $\tilde{\nu}$ being 'pure' monetary policy shocks.

[44] See, e.g., Fabio Panetta, member of the executive board of the ECB, who stated: "Bad inflation reflects negative supply shocks that raise prices and depress economic activity, which monetary policy should look through." (Panetta, 2022)

[45] Once all firms have invested in price flexibility, parameter changes may reduce flexibility but cannot increase it beyond this level.

price flexibility and the pass-through of shocks to inflation. Given that the derivative of the term $\varphi(z)\omega/\Delta$ in the expression for the price index (21) with respect to z is positive, we directly obtain the following lemma.

Lemma 1 (Effect of price flexibility) *A higher price flexibility (a higher z) translates into a larger pass-through of shocks to prices.*

The following proposition then follows from equation (22).[46]

Proposition 1 (Effect of inflation volatility) *Higher inflation volatility leads to higher price flexibility and hence a stronger pass-through of all shocks to prices.*

Intuitively, when competitors' prices fluctuate significantly, firms gain from investing in the ability to adjust their prices in response to the resulting demand shifts. Additionally, increases in cost or demand variability raise inflation volatility and, at the same time, make price adjustments after shocks more advantageous. Enhanced price flexibility, in turn, amplifies the sensitivity of prices to shocks. This mechanism aligns with our empirical finding: higher inflation volatility leads to a stronger pass-through of cost shocks to prices.

Technically, equation (21) implies a larger shock pass-through if more firms have invested in price flexibility (how many firms are able to adjust their price after observing the shocks) and if monetary policy is less aggressive in fighting inflation (by how much do the adjusters adjust). The latter, direct effect of monetary policy on demand is standard in the literature. In particular, a higher value of ϕ raises Δ and corresponds to stricter inflation targeting. In the extreme, ϕ approaches infinity, which fixes the price level at its previous level. Additionally, the impact of ϕ on the variances of the price level and hence demand changes the firms' incentives to invest in price flexibility (see above), which entails an indirect influence of monetary policy via $\varphi(z)$. Regarding the effects of monetary policy, we can derive the following result.

Proposition 2 (Effects of monetary policy) *Stricter inflation targeting (a higher ϕ) reduces the response of inflation to all shocks in two ways: directly by reacting to the change in inflation and indirectly by reducing price flexibility. In contrast, an accommodating monetary policy stance towards supply shocks (raising $Cov(mc, \hat{\nu})$) increases price flexibility (z) and thereby the pass-through of all shocks to prices.*

Regarding the last part of the proposition, note that contractionary supply shocks increase costs and the general price level simultaneously. Seen from the perspective of an individual firm under strategic complementarity in pricing, both developments create an incentive to raise prices.[47] Similar reasoning applies to expansionary demand shocks, which increase demand and the price level. Firms are thus more likely to invest in price

[46]Proofs for the propositions are given in Appendix D.

[47]Strategic complementarity is the standard case in this kind of model and is given by assuming $\alpha < 1$.

flexibility if the correlation of shocks with the price level is high. By dampening the price response, monetary policy can reduce this incentive.[48] A more accommodating policy, overall or just in case of supply shocks, counteracts this reasoning and leads—ceteris paribus—to a higher price flexibility and a higher pass-through of shocks to inflation.

Despite this result, two caveats are in order. First, one argument for a muted monetary policy reaction to supply shocks is their transitory nature in combination with lags in the transmission of policy actions, see Conrad et al. (2025). Given that we consider a stylized model, we do not capture this notion here. Second, we are only interested in the connection between shocks and inflation and, hence, do not conduct a proper welfare analysis.[49]

5.2 Dynamic model

To obtain quantitative predictions beyond those of the analytical one-period version above, we now move on to a numerical simulation of the infinite-period version. Here, we follow Kimura and Kurozumi (2010), which is based on concepts from Devereux and Yetman (2002), and let firms choose their individual degree of price flexibility (their Calvo parameter θ_j) once, given the parameters and shock variances. That is, they can set the probability of being able to adjust prices. As above, they pay the costs Φ whenever firms get the opportunity to do so, such that higher flexibility entails larger costs.

Specifically, we introduce the above structure of raw material inputs to production into the New Keynesian framework of Kimura and Kurozumi (2010) and use the resulting model to analyze different inflation regimes. Thus, we retain the setup of the analytical model in Section 5.1 but assume an infinite planning horizon and allow the wage to be set in each period. For simplicity, we assume constant returns to scale, $\alpha = 1$, and constant costs of being able to adjust prices, $\Phi(j) = \Phi$.[50] Furthermore, for ease of notation, we define this cost, expressed in prices of the aggregate output good, as $F_t \equiv \Phi MC_t/P_t$.

The firm's profit maximization is equivalent to minimizing its loss from not being able to reset its price. Up to second order, this loss is proportional to (see Walsh, 2003)

$$\mathcal{L}_t(\theta_t, \theta) = F_t + \min_{p_{j,t}} E_t \sum_{k=0}^{\infty} (\beta\theta_j)^k (p_{j,t} - p^*_{j,t+k})^2 + \beta(1-\theta_j) \sum_{k+1}^{\infty} (\beta\theta_j)^{k-1} E_t \mathcal{L}_{t+k}(\theta_j, \theta),$$

where lower-case letters refer to variables linearized around the flexible-price steady-state, β is the firms' discount factor, and $p^*_{j,t}$ is the price the firm would set if no nominal rigidities were present, which is $p^*_{j,t} = mc_t = \gamma c_{R,t} + (1-\gamma) w_t$. The wage is determined from households' optimization problem and is given by

$$w_t - p_t = \sigma c_t + \zeta l_t = \xi y_t + \zeta \gamma c_{RR,t},$$

[48]Naturally, lower volatility achieved by reducing monetary policy shocks has the same effect.

[49]Bhattarai et al. (2018) study the effect of price flexibility on welfare and find that greater price flexibility often reduces welfare.

[50]That is, fluctuations in aggregate demand affect costs via the wage rather than through decreasing returns to scale, with similar implications.

where $c_{RR,t} = c_{R,t} - w_t$ is again the relative price of raw materials to labor and $\xi = \sigma + \zeta/[1+(1-\theta)\Phi/Y)]$, with Y denoting steady-state output.[51] The desired price is therefore

$$p^*_{j,t} = p_t + \gamma c_{RR,t} + w_t = p_t + \xi x_t, \tag{24}$$

with x_t as the output gap. The optimal price that results from this minimization is

$$p^0_{j,t} = (1-\beta\theta_j)E_t \sum_{k=0}^{\infty} (\beta\theta_j)^k p^*_{j,t+k} = (1-\beta\theta_j)E_t \sum_{k+0}^{\infty} (\beta\theta_j)^k (p_t + \xi x_{t+k}). \tag{25}$$

Following Kimura and Kurozumi (2010), we assume that the firm chooses its individual Calvo parameter θ_j to minimize the unconditional expected loss in profit due to rigid prices, which is

$$E\mathcal{L}_t(\theta_j, \theta) = \frac{1-\beta\theta_j}{1-\beta} \left[F + E \sum_{k=0}^{\infty} (\beta\theta_j)^k (p^0_{j,t} - p^*_{j,t+k})^2 \right],$$

where E is the unconditional expectations operator and F the unconditional expectation of F_t. That is, firms may decide for higher price flexibility (a lower θ_j) if they reckon that it pays off to be able to respond quickly to changing conditions. This is associated with higher costs, as they have to pay the price-setting costs F more often in this case. The first-order condition is then, using (24) and (25),

$$F + \sum_{k=0}^{\infty} (\beta\theta_j)^{k-1} [(k+1)\beta\theta_j - k] V \left[\sum_{h=1}^{k} \pi_{t+h} + \xi x_t - \tilde{\mathcal{L}}_t(\theta_j, \theta) \right] = 0, \tag{26}$$

where V is the unconditional variance and

$$\tilde{\mathcal{L}}_t(\theta_j, \theta) = \sum_{h=1}^{\infty} (\beta\theta_j)^h E_t \pi_{t+h} + \gamma(1-\beta\theta_j) \sum_{h=0}^{\infty} (\beta\theta_j)^h E_t x_{t+h}.$$

We reach an equilibrium if the optimal $\theta_j = \theta$ for each firm j, which yields the standard New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\gamma(1-\theta)(1-\theta\beta)}{\theta} x_t.$$

On the demand side, household optimization results in the dynamic IS equation

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{t+1} - r^*_t)/\sigma,$$

where r^*_t is the natural rate of interest, which is given by

$$r^n_t = -\frac{\sigma\gamma(1+\zeta)}{\xi} E_t \Delta c_{RR,t+1}.$$

Lastly, we assume a Taylor rule for the interest-rate decisions of the central bank

$$i_t = \phi_\pi \pi_t + \phi_x x_t.$$

To obtain a numerical solution, we search for a θ that, once the model is solved for this value and the equilibrium paths are inserted into (26), fulfills that equation for $\theta_j = \theta$.

[51]This expression is derived from the linearization of aggregate labor demand, given the production function (8), in which D_j equals unity if firm j can set its price and $\alpha = 1$.

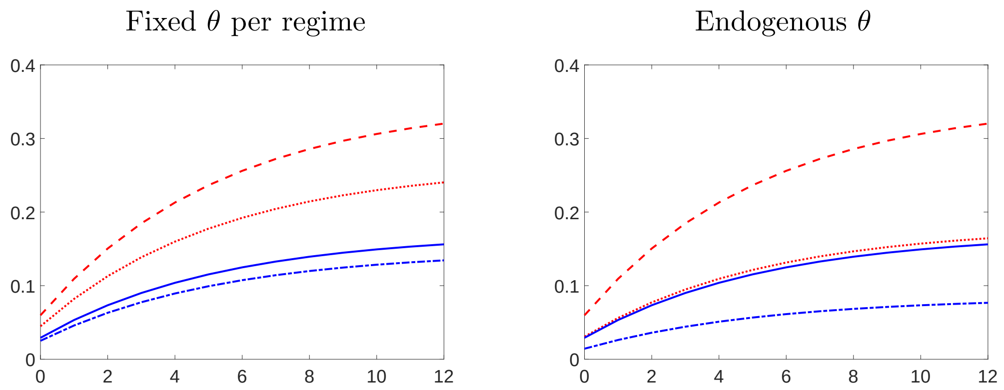

Figure 13: Theoretical responses with counterfactuals. Both panels: Impulse responses in Regime 1 (low volatility, blue solid lines) and Regime 2 (high volatility, red dashed lines) of CPI to a unit shock to the price of crude materials $c_{R,t}$. Left: responses in Regime 1 (blue dashed-dotted line) and Regime 2 (red dotted line) for stricter inflation targeting with unchanged, regime-specific price-setting frequency. Right: responses in Regime 1 (blue dashed-dotted line) and Regime 2 (red dotted line) for stricter inflation targeting with endogenous, regime-specific price-setting frequency. Horizontal axes denote months.

5.2.1 Calibration and model predictions

The calibration of the model equals that of Kimura and Kurozumi (2010), where applicable. That is, we set $\beta = 0.99, \sigma = 1.86, \zeta = 1, \phi_\pi = 1.5, \phi_y = 0.5$ (on an annual basis), and $\rho^n = 0.83$, assuming an AR(1) process for $c_{RR,t}$ and hence r_t^n. Instead of employing their variance of the natural-rate shock across both volatility regimes , however, we set this variance differently in each regime. In particular, we choose values such that the model generates the observed standard deviation of CPI inflation in each regime (0.27% and 0.56%, respectively). The resulting standard deviation of innovations to r_t^n is 0.2% in the low-volatility regime and 0.4% in the high-volatility regime. We then simulate a shock to the real price of raw materials that raises the nominal costs of raw materials by 1%, as in our empirical estimations.

Figure 13 displays the response of CPI inflation after such a shock in the two regimes. The blue solid line in both panels represents the low-volatility scenario with a resulting Calvo parameter of 0.7, while the red dashed line shows the high-volatility case with an endogenous Calvo parameter of 0.53. Considering the stylized structure of the three-equation New Keynesian model, we regard the fit to the corresponding responses in the left panel of Figure 4 as adequately close. In particular and in line with Proposition 1, we obtain a higher inflation on impact and in the following periods in the high-volatility regime, induced by the higher share of price-adjusting firms.

We also conduct two hypothetical scenarios in which the central bank adheres to stricter inflation targeting by increasing its reaction coefficient ϕ from 1.5 to 2. The left panel of Figure 13 shows the responses if we leave the Calvo parameter unchanged for each regime, i.e., at 0.7 and 0.53, respectively. We thereby isolate the traditional monetary-policy channel that reduces inflation by dampening demand. While the stronger reaction already achieves a lower inflation response to the cost-push shock for given values of θ, the effect is magnified once we allow for an endogenous re-adjustment of the price-setting frequency, as also discussed by Kimura and Kurozumi (2010). The right panel displays the corresponding responses that result from optimally chosen Calvo parameters, based on the shock variances in both regimes (which are unchanged) and the new value for ϕ. Specifically, we obtain values of $\theta = 0.79$ in the low-volatility regime and $\theta = 0.67$ in the high-volatility regime. By comparing both panels, we find that the dampening of the inflation response is particularly successful in the high-volatility regime. In particular, the response in the high-volatility regime for $\phi = 2$ is similar to that in the low-volatility regime for $\phi = 1.5$. In short, stricter inflation targeting pays off double in terms of reducing inflation fluctuations, as predicted by Proposition 2.

6 Conclusion

We examine the impact of supply shocks on consumer price inflation in the United States, taking into account different inflation regimes. Employing a Markov-switching model, we identify two distinct regimes and use the filtered state probabilities to construct a regime indicator. It turns out that the regimes are mainly characterized by different inflation volatilities. We then interact a local projections model with the indicator and estimate responses with an LP-IV approach, using data outliers in the Crude PPI series as instruments for supply shocks.

We find that the impulse responses of the CPI following a supply shock are indeed regime-dependent. If a supply shock occurs during the high volatility regime, the increase in consumer prices is more pronounced on impact and takes longer to decay than in times of stable and low inflation. This distinction is not observable when considering different levels of inflation or shock sizes.

The main policy implication we draw from our results for inflation-targeting central banks is that they should pay close attention to the current and potential future inflation regimes when assessing the impact of current developments. If these developments lead to high CPI volatility, the economy may transition to a regime where cost shocks are passed on to consumer prices more rapidly and to a larger extent. This could result in persistently higher CPI inflation volatility. Put differently, a stricter monetary policy stabilizes inflation not only directly, but also indirectly by reducing price flexibility.

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Appendix

A Data description

Seasonally adjusted data on the CPI and the three producer price indices were obtained from the US Bureau of Labor Statistics (BLS). Until 2014, the BLS used the stage of processing (SOP) aggregation system to report producer prices. Afterward, the BLS switched to the Final Demand-Intermediate Demand (FD-ID) system. Table A-1 reports the SOP and the corresponding FD-ID codes as well as the respective variable names.

The BLS defines crude materials as unprocessed goods and intermediate materials as processed goods that businesses purchase as inputs for their production. Products included in the Crude PPI enter the market for the first time and will undergo processing when purchased. In 2024, the index for unprocessed goods for intermediate demand (ID62) was made up of unprocessed foodstuffs and feedstuffs (39%), unprocessed nonfood materials except fuel (49%), and unprocessed fuel (12%). Intermediate materials are already processed to some degree but need further processing before becoming a finished good. Finished goods comprise commodities used for personal consumption or that businesses use as capital investment. Government purchases and exports are excluded from the SOP system.

Seasonally adjusted data on the stages of processing industrial production indices and overall industrial production were retrieved from the Federal Reserve Board (FRB). The indices are classified into raw materials, primary & semifinished processing, and finished processing, and are available since 1972, or 1947 in the case of IP Materials.

SOP Code	Title	FD-ID Code	Title
SOP1000	Crude materials	ID62	Unprocessed goods for intermediate demand
SOP2000	Intermediate materials, supplies and components	ID61	Processed goods for intermediate demand
SOP3000	Finished goods	FD49207	Finished goods

Table A-1: Variable description of Crude (SOP1000), Intermediate (SOP2000), and Finished (SOP3000) PPI. More information available on https://www.bls.gov/ppi/fd-id/ppi-stage-of-processing-to-final-demand-intermediate-demand-aggregation-system-index-concordance-table.htm.

B Econometric checks

Our instrumental variable consists of few non-zero data points and can thus be characterized as a *sparse instrument*. Giacomini et al. (2022) argue that sparse instruments, often constructed from narrative restrictions, are likely to be weak instruments. We test the relevance of our IV by applying the robust test for weak instruments with multiple

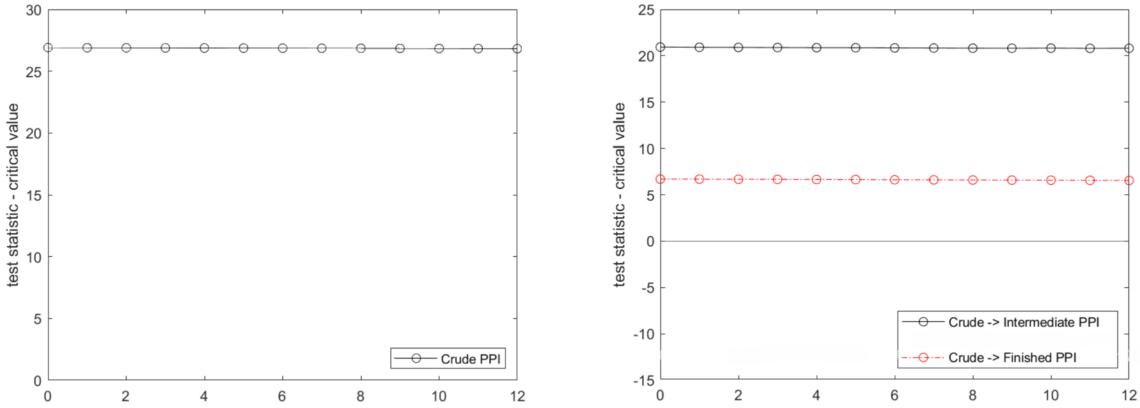

Figure B-1: Left panel: Results of the Lewis and Mertens (2022)-test for weak instruments: difference of test statistic and critical value for baseline results (Figure 4). Right panel: same statistic between stages of processing (Figure 5). Horizontal axes denote months.

endogenous regressors proposed by Lewis and Mertens (2022). We interact the instrument and Crude PPI (our endogenous regressor) with the state indicator H_t and use the same set of controls as in our respective local projection specifications. Following Lewis and Mertens (2022), the test rejects weak instruments if the test statistic lies above the critical value. For our baseline specification, this is the case at all horizons and for all three stages of processing PPIs, as can be seen in Figure B-1.

C Alternative channels

Figure C-1 compares Google searches for the term 'Price escalation clause' with the level of CPI inflation. Despite a broad co-movement, Google searches are much more aligned with the change in the inflation rate, see Figure 1 in the introduction.

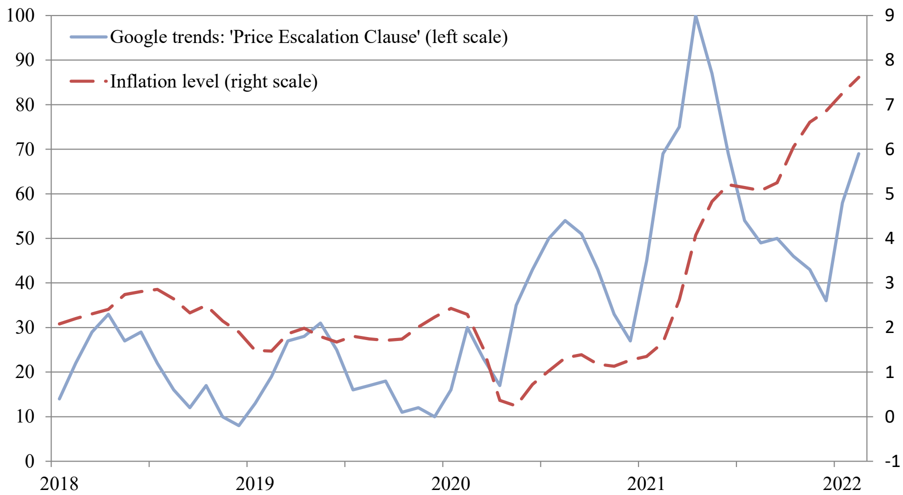

Figure C-1: Escalation clauses. Index for Google searches of 'Price escalation clause' (left axis) and annualized s.a. CPI inflation rate in percent (right scale).

Inflation level above/below:

70th percentile 5 percent

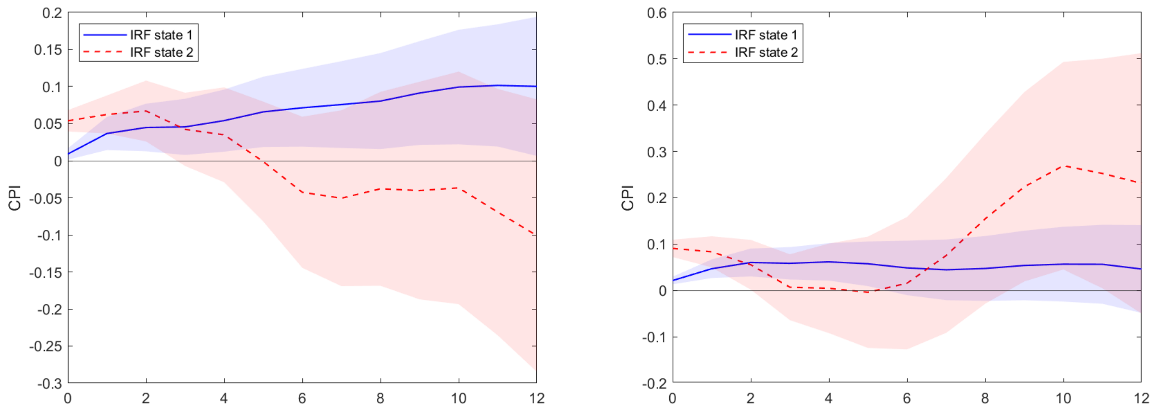

Figure C-2: Different cutoffs for the inflation level. Impulse responses in Regime 1 (low inflation, solid blue lines) and Regime 2 (high inflation, dashed red lines) of CPI to a shock to Crude PPI. Left: state 1/2 if level CPI inflation is below/above its 70th percentile. Right: state 1/2 if CPI inflation is below/above 5%. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

The left panel of Figure C-2 shows the CPI response to supply shocks for regimes if the level of inflation is below (State 1, solid blue lines) or above (State 2, red dashed lines) its 70th percentile. The right panel repeats this exercise with a cutoff at an inflation level of 5%. Overall, we find little evidence of strong state dependency on the inflation level. Although the high-inflation regime shows a significantly larger initial response in both panels, this significance vanishes quickly: the point estimate dips below that of the low-inflation regime and then rises above it in the right panel, but neither deviation is statistically significant.

FFR response EER response

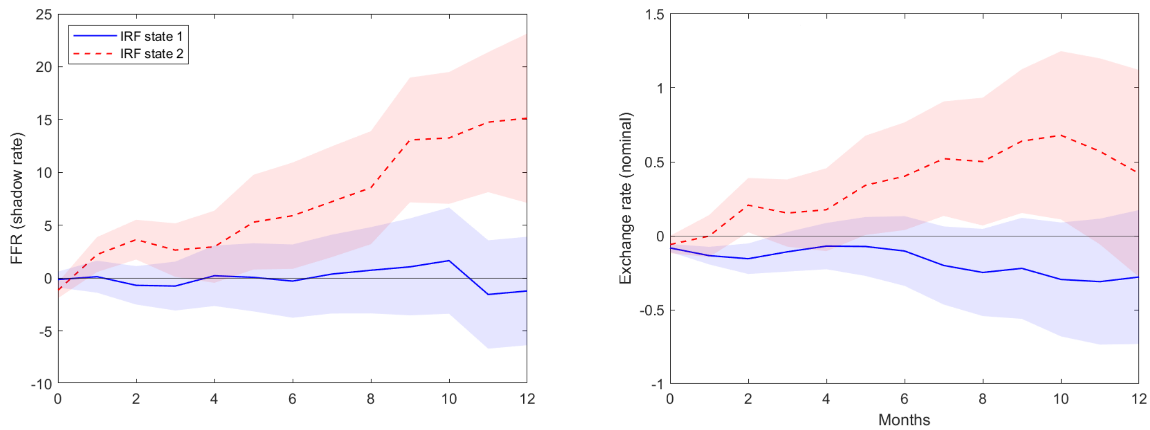

Figure C-3: Impulse responses of shadow rate (left panel) and nominal effective exchange rate (right panel, higher values correspond to an appreciation) in Regime 1 (low volatility, solid blue lines) and Regime 2 (high volatility, dashed red lines) to shocks to PPI. Horizontal axes denote months. Shaded areas represent 68% confidence intervals.

In the left panel of Figure C-3 we depict the response of the shadow rate—the updated series from Krippner (2013)—to shocks to Crude PPI. As visible, the monetary policy reaction is not responsible for the observed state dependency of CPI responses. Monetary policy reacts more to shocks to Crude PPI in State 2 than in State 1, in line with the stronger inflation response. That is, if anything, monetary policy dampens the further course of inflation.

The right panel of the same figure shows the response of the nominal broad effective exchange rate (EER, where increases represent appreciations), provided by the BIS. We reduced the lag number to 8 since the exchange rate is only available from 1994 onward. The exchange rate appreciates more in the high-volatility regime, in line with the stronger interest-rate response. That is, the stronger inflation reaction in the high-volatility regime cannot be explained by a depreciation that leads to rising PPIs at all stages of production and the CPI. Similarly, the responses of the Intermediate PPI and the Finished PPI in Figure 5, which exclude imports, further demonstrate that our results are not driven by the exchange-rate response.

D Model derivations and proofs

Derivation of equation (21). The linearized price index is, see Devereux (2006),

$$p = \varphi(z)p^1,$$

with

$$\varphi(z) = \frac{z\exp(E\ln P^1(1-\varepsilon))}{z\exp(E\ln P^1(1-\varepsilon)) + (1-z)\exp(E\ln P^0(1-\varepsilon))}.$$

The linearized price (18) of flexible firms p^1 reads as

$$p^1 = \alpha\omega mc + (1-\alpha)(\varepsilon-\phi-1)\omega p + (1-\alpha)\omega(\hat{\nu}-\hat{\chi})$$

such that (21) results.

Derivation of equations (22) **and** (23). Given the expression (21) for the price index, we obtain $\hat{y}$ as

$$\begin{aligned}\hat{y} &= \frac{(\varepsilon-\phi-1)\varphi(z)\omega}{\Delta}\left[\alpha mc + (1-\alpha)(\hat{\nu}-\hat{\chi})\right] + \hat{\nu}-\hat{\chi} \\ &= \frac{(\varepsilon-\phi-1)\varphi(z)\omega\alpha}{\Delta} mc + \frac{1}{\Delta}\left(\hat{\nu}-\hat{\chi}\right).\end{aligned}$$

We therefore get the following

$$mc + \frac{1-\alpha}{\alpha}\hat{y} = \frac{1}{\Delta}\left[mc + \frac{1-\alpha}{\alpha}\left(\hat{\nu}-\hat{\chi}\right)\right].$$

The resulting variance is then

$$Var\left(mc + \frac{1-\alpha}{\alpha}\hat{y}\right) = \frac{1}{\Delta^2}\left[\sigma^2_{mc} + \left(\frac{1-\alpha}{\alpha}\right)^2 (\sigma^2_{\hat{\nu}} + \sigma^2_{\hat{\chi}}) + 2\frac{1-\alpha}{\alpha}\sigma_{mc,\hat{\nu}}\right],$$

which can be used in equation (13), together with equation (12), to derive conditions (22) and (23).

Proof of Proposition 1. Note that

$$\begin{aligned}\Delta &= \frac{\alpha + \varepsilon(1-\alpha) - \varphi(z)(1-\alpha)(\varepsilon - \phi - 1)}{\alpha + \varepsilon(1-\alpha)} \\ &= \frac{\alpha - \varphi(z)(1-\alpha)(\phi - 1) + \varepsilon(1-\alpha)(1-\varphi(z))}{\alpha + \varepsilon(1-\alpha)} > 0,\end{aligned}$$

which holds since $\phi < 1$. Furthermore, $\Delta = 1$ at $z = 0$, such that the left-hand-side of inequality (22) is positive at $z = 0$. At this point, the right-hand side $\Phi(0)$ is zero (there is a firm that has zero costs of investing in price flexibility). Moreover, $\Phi'(z) > 0$. The sign of the slope of the left-hand side is determined by

$$\frac{\partial \Delta^{-2}}{\partial z} = 2\Delta^{-3}\omega(1-\alpha)(\varepsilon - \phi - 1)\varphi'(z).$$

This expression is positive if $\phi > 1 - \varepsilon$ and vice versa. A positive slope corresponds to strategic complementarity in the choice of flexibility: the more firms choose to invest in price flexibility, the more it pays off for an individual firm to also do so. A negative slope corresponds to strategic substitutability in the choice of flexibility, see Devereux (2006). We hence get a unique equilibrium value for z if $\phi \leq 1-\varepsilon$. Note that the second derivative of Δ^{-2} with respect to z can only be negative if the first derivative is also negative. For $\phi > 1 - \varepsilon$, we have three possibilities: a) one unique equilibrium at $0 < z < 1$, b) one unique equilibrium at $z = 1$, or c) three equilibria, one for a low value of $0 < z < 1$, one at an intermediate value of $0 < z < 1$, and one at $z = 1$. All considered equilibria are stable—except for the intermediate one in the case of three equilibria—as for lower z the benefit of investing in price flexibility (left-hand side of inequality (22)) is higher than the costs $\Phi(z)$. We therefore disregard the intermediate equilibrium in the case of three equilibria. If we are already at the corner solution, z can not rise any further. Since the left-hand-side of inequality (22), for any given value of z, is increasing in $\sigma^2_{mc}, \sigma_{\hat{\chi}}$, and $\sigma^2_{\hat{\nu}}$, and its slope is, for interior solutions, larger than that of the right-hand-side, the following statement regarding the impact of the shock variances obtains: higher volatilities of the shocks to the costs of raw materials ($\sigma^2_{c_R}$), demand ($\sigma^2_{\hat{\chi}}$), and/or the money supply ($\sigma^2_{\hat{\nu}}$, for a given covariance with input costs) raise inflation volatility, price flexibility (z), and hence the pass-through of shocks to prices.

In the last step, we consider the general-equilibrium level of the price level (21),

$$\begin{aligned}\sigma_p^2 &= \left(\frac{\varphi(z)\omega}{\Delta}\right)^2 \left[\alpha^2\sigma_{mc}^2 + (1-\alpha)^2(\sigma_{\hat{\nu}}^2 + \sigma\hat{\chi}^2) + \alpha(1-\alpha)\sigma_{mc,\hat{\nu}}\right] \\ &= \frac{2\alpha(\varphi(z)\omega)^2}{\Omega}\Delta(\Theta).\end{aligned}$$

The proposition then directly follows from this. ■

Proof of Proposition 2. The direct effect of a changing ϕ is visible when taking the derivative with respect to ϕ of the term in the price index (21) that multiplies all shocks:

$$\frac{\partial\varphi^2(z)\omega\Delta^{-1}}{\partial\phi} = \varphi(z)\omega\Delta^{-2}\omega(1-\alpha) > 0.$$

Reducing ϕ (stricter inflation targeting) hence decreases the effect of shocks on inflation for a given value of z. The indirect effect of changing ϕ on z depends on the following derivatives (remember that $\Delta > 0$ from the proof of Proposition 1):

$$\begin{aligned}\frac{\partial\Delta^{-2}}{\partial\phi} &= 2\Delta^{-3}\varphi(z)\omega(1-\alpha) \geq 0 \\ \frac{\partial\Delta(\Theta)}{\partial\sigma_{mc,\hat{\nu}}} &= \frac{\Omega(1-\alpha)}{\Delta^2} > 0,\end{aligned}$$

where the first derivative determines the sign of $\partial\Delta(\Theta)/\partial\phi$ and $\Delta(\Theta)$ is the left-hand-side of inequality (22). Proposition 2 follows directly from these derivatives. ■