Implications of Inflation Forecast Targeting for Fiscal Policy*

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Abstract

Many central banks link policy decisions to inflation forecasts, a feature absent from standard New Keynesian models. We examine equilibrium determinacy and monetary–fiscal interactions under inflation forecast targeting. Determinacy requires the monetary policy response to expected inflation to satisfy not only the Taylor principle but also an upper bound, which depends on the Phillips curve slope and tightens with the forecast horizon. Longer horizons thus first make welfare-maximizing policy incompatible with determinacy and eventually preclude determinacy entirely. Fiscal policy can restore determinacy only by abandoning active debt stabilization. Inflation forecast targeting may therefore induce monetary passivity and invite fiscal dominance.

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1 Introduction

Modern monetary policy is conducted with an explicit awareness of transmission lags. Central banks adjust policy rates today in anticipation of macroeconomic conditions several quarters, often years, into the future. In particular, inflation forecast targeting at one- to three-year horizons plays a crucial role in policy deliberations, as reflected in central bank forecasts, forward guidance, and communication strategies. As Schnabel (2024) notes, the ECB's policy-relevant horizon is usually two to three years. The underlying premise is that changes in the policy rate affect inflation only with a substantial lag, so stabilizing inflation requires responding to expected future inflation rather than to current realizations.[1]

Standard New Keynesian models, however, abstract from this feature in a crucial way. Monetary policy is typically modeled using a Taylor rule, in which the policy rate responds to contemporaneous inflation or, at most, to expected inflation one period ahead.[2] Monetary policy, hence, reacts to inflation outcomes at horizons shorter than those emphasized in actual policy practice. This paper argues that this discrepancy has important implications for equilibrium determinacy, welfare, and the monetary-fiscal interactions. Once the Taylor rule is modified to respond to inflation forecasts several periods ahead, at the so-called target horizon, the central bank no longer reacts directly to the inflation rate it can influence most effectively. Instead, monetary policy affects short-run inflation through current demand conditions while responding to inflation further in the future. This separation weakens the link between short-run inflation movements and the interest rate, creating additional scope for self-fulfilling fluctuations in expectations. This mechanism fundamentally changes the conditions for determinacy and, therefore, the constraints for monetary and fiscal policy. Besides the familiar lower bound implied by the Taylor principle, inflation forecast targeting introduces an upper bound on the monetary policy response. The intuition behind this upper bound lies in the differential response of inflation across horizons. As is commonly known, an economic system is indeterminate if a non-fundamental increase in inflation lowers the real interest rate, thereby confirming the increase. Such an expansionary monetary-policy response can result if the Taylor principle is violated, but also if a sunspot shock pushes up current inflation while it is expected to be low at the target horizon.[3] A strong monetary policy

[1] The theoretical underpinnings of this policy go back to Svensson (1997) and Svensson (1999).

[2] Other versions include longer forecast horizons, but assume that central banks react to the average inflation between now and this horizon, see, e.g., Clarida et al. (1998, 2000) and Enders et al. (2013). We discuss this version in an extension. It is, however, not the current policy of the ECB, as it explicitly aims to 'look through' short-term inflation movements.

[3] Take December 2021, when inflation was running at 5%, as an example for a situation of current high and expected low inflation. ECB president Christine Lagarde explained the decision for expansionary policy as follows: "Our new staff projections foresee annual inflation at ... 3.2 per cent in 2022, 1.8 per cent in 2023, and 1.8 per cent in 2024.... Monetary accommodation is still needed ... for inflation to stabilise at our two per cent inflation target over the medium term."

response to inflation at the target horizon—that is, a *high* Taylor-rule coefficient—amplifies this expansionary response. Low interest rates translate, via stronger economic activity, into high current inflation if the Phillips curve is sufficiently steep. A longer target horizon, in turn, allows more time for inflation to decline from its currently elevated level to the lower rate expected at the target horizon. Indeterminacy therefore arises from a combination of a high Taylor-rule coefficient, a steep Phillips curve, and a long target horizon. We conclude that monetary policy should therefore neither react too weakly to strong inflation (the lower bound due to the Taylor principle) nor too strongly to weak expected inflation (the upper bound).

The additional constraint on monetary policy has important implications for welfare. We show that the welfare-optimal response to expected inflation becomes more aggressive as the forecast horizon increases. This creates a tension with equilibrium determinacy: while longer forecast horizons raise the welfare-optimal Taylor coefficient, they simultaneously tighten the upper bound required for determinacy. Beyond a critical forecast horizon, the welfare-optimal policy rule therefore lies outside the determinacy region and can no longer be implemented as a unique rational expectations equilibrium. The extent of this trade-off again depends on the slope of the Phillips curve: a flatter Phillips curve enlarges the determinacy region and allows the welfare-optimal rule to remain implementable at longer horizons.

Importantly, our findings also change the role of fiscal policy. When monetary policy cannot ensure determinacy on its own, an additional nominal anchor is required. We show that introducing fiscal policy into this environment restores determinacy for a specific subset of the indeterminacy region in a robust and economically meaningful way. When fiscal policy is specified such that it does *not* actively stabilize government debt, and monetary policy is modeled passively in the classic Leeper (1991) sense, the price level is pinned down by fiscal considerations. In this regime, fiscal policy acts as the nominal anchor, and equilibrium uniqueness is restored for any target horizon. Non-fundamental fluctuations in inflation would then lead to explosive government debt, which is ruled out. Our results therefore reveal a novel policy trade-off. If inflation is targeted sufficiently far into the future and monetary transmission is sufficiently strong, policymakers may face a choice between accepting equilibrium indeterminacy and adopting an active-fiscal/passive-monetary regime. Consequently, inflation forecast targeting may unintentionally invite fiscal dominance, not because the central bank deliberately weakens its response to inflation, but because the chosen monetary policy framework no longer admits a determinate active monetary equilibrium. More generally, our analysis shows that the design of the monetary policy rule is itself a determinant of the monetary-fiscal policy regime, since the interaction between monetary and fiscal policy also depends on the time horizon over which monetary policy is conducted.

These insights change the conventional classification of monetary and fiscal policy regimes. In the standard framework of Leeper (1991), monetary policy is characterized solely by the size of the inflation feedback coefficient. Under inflation forecast targeting, this characterization becomes incomplete. Whether a given Taylor rule delivers determinacy depends not only on the strength of the policy response but also on the forecasting horizon and the structural transmission of monetary policy, summarized by the slope of the Phillips curve. Monetary policy is therefore active only if the policy response is sufficiently strong, but not excessively aggressive, the Phillips curve is fairly flat, and the forecast horizon is not too long.

We further show that the introduction of transmission lags in monetary policy actions, often used to justify inflation forecast targeting, does not alter our main conclusions. It mitigates, but does not eliminate, the described mechanisms. The same is true for conventional anchors in the Taylor rule. Allowing the policy rate to respond to the output gap or to its own lagged value may restore monetary dominance. However, these anchors must become stronger as forward-lookingness increases. For long policy horizons, a sufficiently steep Phillips curve, and passive fiscal policy, only a strongly anchored Taylor rule can support regime switching from fiscal to monetary dominance by altering the reaction to inflation.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 provides the intuition for our main results, before Section 4 introduces the baseline New Keynesian model. Section 5 formally analyzes equilibrium determinacy under a purely forward-looking Taylor rule. Section 6 demonstrates that introducing fiscal policy restores equilibrium determinacy for any target horizon. Section 7 considers extensions of the baseline framework, and Section 8 concludes.

2 Related literature

This paper contributes to three strands of literature. First, it relates to work on inflation forecast targeting and forward-looking monetary policy rules. Early contributions by Svensson (1997) and Svensson (1999) develop the conceptual framework of inflation forecast targeting, emphasizing that monetary policy should respond to projected future inflation rather than current realizations to account for transmission lags. His work, however, is neither embedded in a microfounded New Keynesian framework nor does he consider longer forecasting horizons, such that indeterminacy issues do not arise. Subsequent policy-oriented and empirical work documents the prominence of forecast-based decision-making in practice and discusses its implications for policy design and communication (see, among others, Haldane 1997, and Clinton et al. 2015). More recently, Adrian et al. (2018) provide a comprehensive overview of inflation forecast targeting as a guiding principle for modern monetary policy, both in

advanced and emerging economies. Conrad et al. (2025) show that central bank forecasts used for inflation forecast targeting are uninformative beyond a horizon of one year. On the theoretical side, several papers incorporate explicit forecast horizons into Taylor-type policy rules. Batini and Haldane (1999) and Conrad et al. (2025) formalize monetary policy rules that respond to medium-term inflation forecasts and analyze their macroeconomic implications within New Keynesian frameworks. The present paper relates to this line of work by focusing on purely forward-looking Taylor rules and their implications for equilibrium determinacy and fiscal policy.

Second, the paper contributes to the literature on equilibrium determinacy and sunspot fluctuations in New Keynesian models. A large body of work has shown that indeterminacy typically arises when monetary policy reacts too weakly to inflation, violating the Taylor principle. Classic contributions such as Bernanke and Woodford (1997), Clarida et al. (2000), Woodford (1994), Woodford (2003, Chapter 4), and Carlstrom and Fuerst (2002) emphasize that an insufficiently aggressive inflation response allows self-fulfilling fluctuations and sunspot equilibria to emerge. Relatedly, Benhabib et al. (2001) highlights how certain policy rules can give rise to multiple equilibria even in otherwise standard monetary models. A closely related strand of the literature studies the determinacy properties of forward-looking monetary policy rules. Levin et al. (2003) show that when monetary policy responds to inflation forecasts at longer horizons and lacks interest rate smoothing, equilibrium determinacy may fail once the target horizon exceeds one period. Woodford (2000) and Svensson and Woodford (2004) find comparable results. Similar to Bernanke and Woodford (1997), Giannoni and Woodford (2003) note that excessively strong responses to sufficiently distant inflation forecasts can lead to indeterminacy in their analysis of optimal interest-rate rules. Batini and Pearlman (2002) study the interaction between the feedback horizon and the strength of the policy response and derive a sufficient condition under which determinacy is not attainable, depending on the target horizon. More recently, Loisel (2026) characterizes the determinacy properties of stabilization rules. His analysis is very general, but also includes a forward-looking Taylor rule. While these contributions focus on the general possibility of indeterminacy, we derive in detail the upper bound on the Taylor coefficient for higher target horizons, provide economic intuition for its existence, and explore its dependence on the target horizon and the slope of the Phillips curve in the standard New Keynesian model. This lets us determine exactly when indeterminacy arises, rather than providing only a sufficient condition. Our analysis also differs from this literature by explicitly considering the fiscal-policy implications of the monetary determinacy problem. While Loisel (2026) allows for forward-looking fiscal policy, he does not analyze how fiscal policy can restore determinacy when monetary policy is overly forward-looking. Specifically, under sufficiently

forward-looking inflation targeting, there may exist no active monetary policy equilibrium. In this case, equilibrium determinacy requires an active fiscal authority, implying that inflation forecast targeting can endogenously induce fiscal dominance.

Another related strand of the literature emphasizes that the Taylor principle is not universal but depends on the economy's structural characteristics. Galí et al. (2004) show that the presence of rule-of-thumb consumers can alter the design of optimal interest-rate rules. Building on this insight, Bilbiie (2008) demonstrates that limited asset market participation may overturn the conventional aggregate demand logic, implying that equilibrium determinacy can require a passive monetary policy ("inverted Taylor principle"). Bilbiie and Straub (2013) further characterize the conditions under which this reversal arises. In contrast to this literature, our results do not rely on household heterogeneity or limited asset market participation. Instead, we show that even in an otherwise standard New Keynesian framework, the forward-looking horizon of the monetary policy rule fundamentally changes the determinacy conditions and how these conditions crucially depend on the economy's fundamentals, such as the slope of the Phillips curve.

Third, the paper connects to the active–passive monetary–fiscal policy literature initiated by Leeper (1991). In his framework, equilibrium determinacy depends on whether monetary or fiscal policy provides the nominal anchor, with monetary passiveness typically arising when the inflation response in the Taylor rule is too weak to stabilize prices. Closely related is the Fiscal Theory of the Price Level, which emphasizes the role of fiscal behavior and nominal government debt valuation in determining the price level, see, among others, Sims (1994), Woodford (1995), Woodford (2001), Cochrane (1998), Cochrane (2001), and Cochrane (2023). Within this literature, price-level determination follows from the intertemporal government budget constraint, rather than from monetary control of inflation alone. Recent work by Rachel and Ravn (2025) embeds these fiscal valuation effects into a New Keynesian framework with nominal government debt. We present an alternative channel through which monetary policy must be passive to achieve determinacy: the forward-looking nature of the policy rule. Furthermore, the paper contributes to the active-passive literature by highlighting an additional parameter restriction, beyond the Taylor coefficient, that affects whether monetary policy is effectively active under purely forward-looking rules, involving the slope of the New Keynesian Phillips curve. When inflation responds strongly to real activity, achieving determinacy is more difficult.

3 Forecast targeting, indeterminacy, and fiscal policy

Before turning to the formal model in the following section, we first discuss our main results in intuitive terms. We start by arguing why inflation forecast targeting can lead to indeterminacy, and then show how an active fiscal policy can restore determinacy.

3.1 Indeterminacy under inflation forecast targeting

Generally speaking, indeterminacy arises when expectations about future developments cause precisely these developments. That is, expectations become self-fulfilling. Applied to monetary policy, this implies that expectations of higher inflation do not prompt the central bank to raise the nominal interest rate sufficiently to increase the real interest rate. If current inflation is *positively* correlated with inflation at the expectation horizon, this failure raises inflation today and hence at that horizon, validating the initial beliefs.[4] This is the intuition for the well-known Taylor principle. Technically, a too-low inflation-reaction coefficient ϕ_π in the Taylor rule lets one eigenvalue of the system fall below unity, so the system returns to its steady state after potential sunspots.

A second possibility arises for inflation forecast targeting if current inflation is *negatively* correlated with inflation at the monetary policy target horizon. In this case, speculation about a low rate at this horizon triggers an expansionary interest-rate response, pushing up current inflation. Because of the negative correlation, it then falls towards the expected low levels, confirming the initial beliefs. Three parameters are crucial for this outcome. First, a high value of ϕ_π induces a substantially lower interest rate in response to the expected decline in inflation. Second, a steep Phillips curve lets the resulting economic expansion exert a strong upward effect on current inflation. This output-gap-driven inflationary pressure overturns the negative influence of expected low future inflation via the New Keynesian Phillips Curve. High current inflation can therefore coexist with low expected inflation, validating the initial expectations. Third, a longer target horizon gives inflation more time to decline from its current high level to the lower level expected at that horizon. Technically, this second possibility of indeterminacy can occur in two ways: either one eigenvalue of the system is inside the unit circle and negative, leading to shorter alternations between high and low inflation, or two complex-conjugate eigenvalues are inside the unit circle. In this case, sunspots trigger slow-moving cycles.

For inflation forecast targeting, there is hence an upper bound on the interest response to inflation to remain within the determinacy region, in addition to the 'traditional' lower bound of the Taylor principle. This also implies that the welfare-maximizing Taylor-rule

[4]For simplicity we assume no transmission lags here, but discuss them further below.

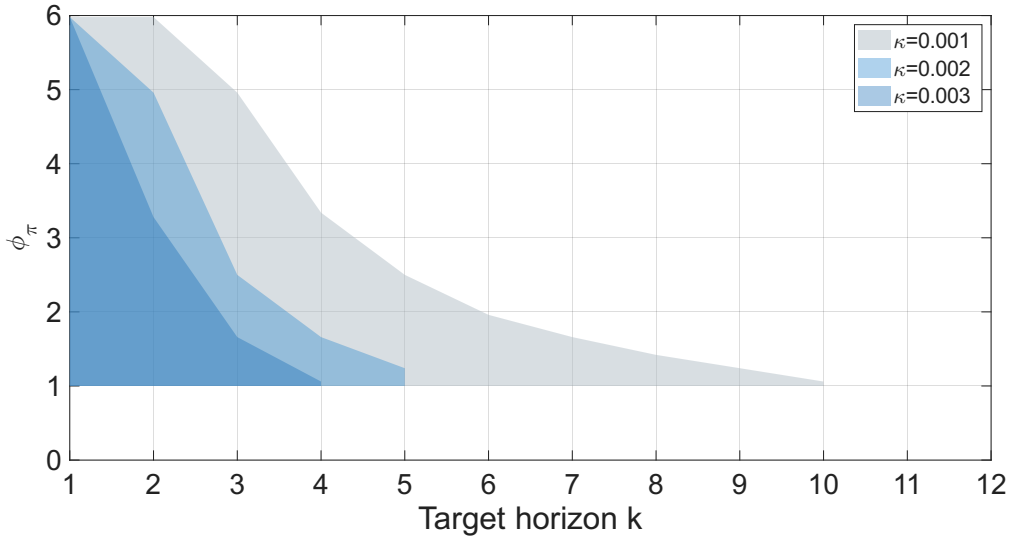


Figure 1: Determinacy region as a function of target horizon k and policy parameter ϕ_π for different values of κ with $\beta = 0.99$, based on the model of Section 4.

coefficient falls outside the determinacy region as the target horizon increases, as we show below. Moreover, raising the target horizon further causes indeterminacy quickly for any Taylor-rule coefficient with plausible values of the target horizon and κ.

Figure 1 illustrates the determinacy regions for ϕ_π as a function of the target horizon k for different values of the slope of the Phillips Curve κ in our baseline model of Section 4. Two observations stand out. First, the determinacy region shrinks as the target horizon increases. Second, for any given target horizon, the admissible range of Taylor coefficients becomes substantially larger as the Phillips curve flattens. Thus, a smaller value of κ weakens the mechanism underlying indeterminacy and allows monetary policy to remain effective even when responding to inflation forecasts further into the future. Equivalently, for a given target horizon, there exists a maximum admissible Taylor coefficient, $\bar{\phi}_\pi$, which decreases in κ. If $\bar{\phi}$ falls below unity, the lower bound, determinacy can no longer be achieved. For example, when $\kappa = 0.003$, determinacy is attainable only up to $k = 4$, whereas with the flatter Phillips curve ($\kappa = 0.001$), determinacy remains feasible up to $k = 10$. In general, determinacy is possible only for very small values of κ if the target horizon exceeds a few quarters.

3.2 Fiscal-Monetary Interactions

These results have an implication for the conventional classification of monetary and fiscal policy regimes. In the standard framework, monetary policy is typically characterized as active when the inflation feedback coefficient satisfies the Taylor principle, while fiscal policy adjusts to stabilize government debt. In that environment, the policy regime largely depends

on the size of the monetary policy coefficient ϕ_π. Under inflation forecast targeting, however, this characterization becomes incomplete. Because policy responds to expected inflation at horizon k, the transmission from expectations to current inflation depends critically on the slope of the New Keynesian Phillips curve, besides k. As a result, determinacy is no longer governed by ϕ_π alone, but by the interaction between policy and the structural transmission of inflation, summarized by κ. Consequently, in an inflation-forecast-targeting environment with a given target horizon, an active monetary policy regime in the sense of Leeper (1991) requires two conditions to hold simultaneously: the inflation response coefficient must be sufficiently large but not too large, and the Phillips-curve slope must be sufficiently small.

If either of the two conditions fails, fiscal policy can restore determinacy in a specific subset of these regions of indeterminacy. The regions are characterized by whether monetary policy is passive in the conventional Leeper (1991) sense, that is, whether $\phi_\pi < 1$. Consider the example of sunspot-induced higher inflation. If the nominal interest rate moves little, the higher surprise inflation reduces the value of existing government debt. With an active fiscal policy, this surplus would not be used to reduce taxes or increase spending correspondingly in the future, setting government finances on an explosive path. Since we rule out such paths, the sunspot cannot occur in the first place, restoring determinacy. This is the standard analysis of passive monetary and active fiscal policy as in Leeper (1991).

If indeterminacy arises not because the Taylor principle is violated but because ϕ_π is too high, the situation changes fundamentally. As described above, a low real interest rate may be set despite current high inflation since monetary policy responds strongly to low expected future inflation. In this case, the initially high inflation will be followed by low inflation and a corresponding positive real interest rate, which can undo the previous positive effect on government debt. Following a sunspot, the future path of inflation hence adjusts such that government finances remain bounded, even if fiscal policy is active in the standard sense. Active fiscal policy can therefore not rule out sunspots if indeterminacy arises due to large values of ϕ_π.

As shown above, our results imply that for sufficiently large values of κ and/or k, there exists no active monetary policy rule that delivers determinacy. If policymakers nevertheless seek to implement a unique equilibrium, the only remaining possibility is the conventional active-fiscal/passive-monetary regime with $\phi_\pi < 1$. In this sense, inflation forecast targeting can endogenously force monetary policy into a passive role. This observation has important implications for monetary policy, as adopting an inflation-forecast-targeting framework may invite fiscal dominance. Importantly, fiscal dominance is not primarily driven by an insufficient monetary policy response. Instead, it emerges because the chosen monetary policy framework no longer allows an active monetary regime to uniquely determine the equilibrium.

4 Baseline model

This section presents a New Keynesian model with a forward-looking Taylor rule and an explicit fiscal block, which forms the basis for the subsequent determinacy analysis. Households and Firms are modeled as in the textbook by Galí (2008). The linearized IS equation and the New Keynesian Phillips Curve are:

$$y_t = \mathbb{E}_t y_{t+1} - i_t + \mathbb{E}_t \pi_{t+1} \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t, \tag{2}$$

where we omit the deviations of the natural rate of interest from its steady-state value, r_t^n, as we are here not interested in movements due to changes in fundamentals. The key difference in the model setup compared to a more standard version is the purely forward-looking Taylor rule

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+k} + e_t^i, \tag{3}$$

where i_t is the one-period nominal interest rate, ϕ_π denotes the Taylor coefficient, and e_t^i is a monetary policy shock. This Taylor rule captures the idea that central banks, such as the ECB, adjust the policy rate in response to inflation forecasts at horizon $t + k$, rather than to contemporaneous inflation or short-horizon expectations. The standard case in which the central bank targets current or one-period-ahead inflation corresponds to $k \in \{0, 1\}$.

We model fiscal policy using the fiscal block as in Rachel and Ravn (2025). The fiscal authority chooses the primary surplus (equivalently, the deficit) and issues nominal one-period government bonds. Unlike Rachel and Ravn (2025), we abstract from institutional components that do not affect the government budget constraint and focus on the interaction between nominal debt valuation, inflation, and the surplus rule that closes fiscal policy. Let B_t denote the nominal stock of government debt due in period t and P_t the price level. The evolution of nominal government debt can then be written in nominal terms as the present value of newly issued debt being equal to maturing debt net of the primary surplus:

$$\frac{B_t}{1 + i_t} = B_{t-1} - P_t s_t,$$

where s_t is the real primary surplus, with, following Rachel and Ravn (2025), the sign convention that a higher s_t corresponds to a tighter fiscal stance. Define real government debt as $v_t \equiv B_t / P_t$. Using this definition together with inflation $\pi_t \equiv P_t / P_{t-1}$, the budget constraint can be expressed in real terms:

$$\frac{v_t}{1 + i_t} = \frac{v_{t-1}}{\pi_t} - s_t.$$

This representation makes explicit that inflation affects the real value of outstanding nominal liabilities and therefore enters the fiscal transmission channel through debt revaluation. Again following Rachel and Ravn (2025), fiscal policy is closed with a systematic surplus rule that responds to lagged real debt and contemporaneous output, and includes an exogenous surplus disturbance. Concretely, the surplus is specified in levels as a stationary rule around a deterministic steady state:

$$s_t = \bar{s}\left(\frac{v_{t-1}}{\bar{v}}\right)^{\alpha_b}\left(\frac{y_t}{\bar{y}}\right)^{\alpha_y}\exp([1-\alpha_b]e_t^s),$$

where $\bar{s}$ is the steady-state surplus, $\alpha_b \in [0,1)$ governs the feedback from debt to the surplus, α_y captures the automatic response of the surplus to output, and e_t^s is a mean-zero surplus shock. The steady state is chosen such that the surplus rule is consistent with balanced debt dynamics given the steady-state net real interest rate r. Specifically, the steady-state surplus satisfies $\bar{s} = \bar{v}\, r/(1+r)$. We study local dynamics using a first-order approximation around a non-inflationary steady state with $\bar{\pi} = 1$ and zero steady-state government expenditures.[5] We then have $s_t = -\tau_t$, where τ represents real lump-sum taxes. Log-linearizing the real government budget constraint and the surplus rule yields the fiscal block:

$$v_t = (v_{t-1} - s_t)/\beta + \gamma(i_t - \pi_t/\beta) \tag{4}$$

$$s_t = \alpha_b(v_{t-1} - e_t^s) + \alpha_y y_t + e_t^s, \tag{5}$$

where $\gamma = v^{ss}/y^{ss}$ is the steady-state debt-to-GDP ratio. Equation (4) is the linearized law of motion for real government debt. Debt increases with primary deficits and the cost of servicing nominal liabilities, and falls with inflation, which revalues outstanding debt. Equation (5) describes systematic fiscal behavior: α_b controls how strongly the surplus reacts to debt deviations from steady state, while α_y captures automatic stabilizers operating through output. As in Rachel and Ravn (2025), the interaction between monetary and fiscal policy is intentionally asymmetric. Monetary policy affects fiscal dynamics because government debt is denominated in nominal terms. Movements in the nominal interest rate change debt-service costs, while movements in inflation revalue outstanding liabilities in real terms. Both channels enter the linearized government budget constraint through the terms involving i_t and π_t. Fiscal policy then responds to these debt dynamics through the surplus rule, which links primary surpluses to lagged real debt and to the output gap via automatic stabilizers. By contrast, due to the linearization around $\bar{G} = 0$, the optimality conditions of the private

[5] As in Rachel and Ravn (2025), the latter assumption allows us to focus on first-order effects of monetary policy actions on fiscal policy, such that we can isolate the role of monetary and fiscal policy in the determinacy conditions.

sector and the monetary policy rule are specified independently of fiscal variables. Moreover, the central bank sets the nominal interest rate as a function of the inflation forecast at horizon $t + k$, abstracting from fiscal considerations. This specification reflects the institutional separation between a price-stability-oriented central bank and the fiscal authority, and rules out mechanisms such as default risk premia or direct monetary accommodation of public debt. The block-recursive structure also allows the effects of fiscal policy on equilibrium determinacy to be identified transparently. Since the IS equation, the New Keynesian Phillips curve, and the forward-looking Taylor rule do not depend on fiscal variables, monetary policy's ability to anchor inflation can be assessed in isolation.

5 Indeterminacy under a forward-looking Taylor rule

This section analyzes equilibrium determinacy under increasingly forward-looking Taylor rules. We proceed by gradually extending the inflation-targeting horizon, focusing on the monetary policy block consisting of the IS equation, the New Keynesian Phillips curve, and the forward-looking Taylor rule. We begin by establishing that, under the benchmark calibration of the Phillips-curve slope ($\kappa = 0.1$), monetary policy alone cannot ensure determinacy once the central bank targets inflation more than one period ahead.[6] In this case, determinacy breaks down already at short forward-looking horizons and cannot be restored by adjusting the inflation response coefficient. We then show that this result is not universal, but depends critically on the slope of the Phillips curve. In the final part of this section, we characterize a threshold value $\bar{\kappa}(k, \beta, \phi_\pi)$ below which monetary policy can sustain determinacy and above which it cannot.

5.1 The standard case: $\mathbf{k = 0, 1}$

For $k = 1$, the Taylor rule reads

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1} + e_t^i,$$

with e_t^i representing a monetary policy shock. Combining the IS equation and the New Keynesian Phillips curve yields a two-dimensional forward-looking system in (π_t, y_t). Under a one-period-ahead inflation target, equilibrium determinacy obtains under the familiar Taylor principle, $\phi_\pi > 1$, together with an upper-bound restriction implied by the Blanchard–Kahn conditions:

$$1 < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa}. \tag{6}$$

[6] We further assume $\beta = 0.99$ if we refer to the benchmark calibration.

This is the standard determinacy result for forward-looking monetary policy (see, e.g., Woodford 2003), where the additional upper bound is typically fulfilled for standard calibrations of κ between 0.01 and 0.1. That is, the monetary policy block can generate two unstable eigenvalues, matching the two forward-looking variables (the derivations are in Appendix A). Section 3.1 provides the intuition for the upper bound. The bound disappears for $k = 0$, as the central bank directly counteracts contemporaneous (non-fundamental) inflation movements in this case.

5.2 New indeterminacy regions: k = 2

Extending the target horizon to two periods fundamentally alters the model's determinacy properties. For $k = 2$, the nominal interest rate reacts to $\mathbb{E}_t\pi_{t+2}$. Iterating the Phillips curve forward allows us to express the Taylor rule in terms of one-step-ahead inflation and output expectations, such that the resulting system remains two-dimensional. (see Appendix B). Applying the Blanchard–Kahn conditions yields the restriction

$$1 < \phi_\pi < \frac{1-\beta}{\kappa}. \tag{7}$$

Since $(1-\beta)/\kappa < 1$ under standard calibrations, no ϕ_π satisfies both inequalities. The Blanchard–Khan conditions thus fail, except for very low values of κ, as the monetary policy block can generate at most one unstable eigenvalue for two forward-looking variables under our baseline calibration. Monetary policy can no longer guarantee a unique equilibrium. Importantly, this condition suggests that determinacy depends critically on κ. In particular, as for $k = 1$, the upper bound $(1-\beta)/\kappa$ decreases in κ, indicating that smaller values of the Phillips-curve slope expand the range of policy parameters for which determinacy may be attainable.

5.3 Longer target horizons: k > 2

The breakdown of determinacy becomes even more likely for $k > 2$. We first derive the system for an arbitrary target horizon $k > 2$ and then use the analytically tractable case $k = 3$ to characterize the determinacy boundary. As will become clear, this result again depends critically on the slope of the Phillips curve, which governs how strongly forward-looking policy feeds back into current inflation dynamics.

Monetary policy and auxiliary variables. For a general target horizon k, the forward-looking Taylor rule is given by

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+k} + e_t^i. \tag{8}$$

We again aim to express the system in first-order form. To do so, we use the New Keynesian Phillips curve and iterate it $k-1$ periods ahead, expressing $\mathbb{E}_t \pi_{t+k}$ as a linear function of one-period-ahead inflation expectations and expectations of future output up to horizon $k-1$. The generalized Taylor rule becomes:

$$i_t = \frac{\phi_\pi}{\beta^{k-1}} \mathbb{E}_t \pi_{t+1} - \phi_\pi \kappa \sum_{j=1}^{k-1} \frac{1}{\beta^{k-j}} \mathbb{E}_t y_{t+j} + e_t^i \tag{9}$$

To render the system first-order, we introduce a sequence of auxiliary variables

$$f_t^1 \equiv \mathbb{E}_t y_{t+1}, \qquad f_t^j \equiv \mathbb{E}_t f_{t+1}^{j-1} \quad \text{for } j = 2, \ldots, k-2, \tag{10}$$

so that $f_t^j = \mathbb{E}_t y_{t+j}$. Using these definitions and inserting the Taylor rule into the IS equation (1), it can be written as

$$y_t = \left(1 - \frac{\phi_\pi}{\beta^{k-1}}\right) \mathbb{E}_t \pi_{t+1} + \left(1 + \frac{\phi_\pi \kappa}{\beta^{k-1}}\right) f_t^1 + \sum_{j=1}^{k-2} \frac{\phi_\pi \kappa}{\beta^{k-1-j}} \, \mathbb{E}_t f_{t+1}^j - e_t^i. \tag{11}$$

Together with the New Keynesian Phillips curve and the auxiliary equations defining $(f_t^j)_{j=1}^{k-2}$, equation (11) characterizes the non-fiscal block under a forward-looking Taylor rule with target horizon k. The system is now first-order, but at the cost of introducing $k-2$ forward-looking variables in addition to $E_t y_{t+1}$ and $E_t \pi_{t+1}$. Therefore, for the target horizon k, this policy block contains k forward-looking variables and the state vector (containing only jump variables) can be expressed as

$$z_t = \left(\pi_t, y_t, f_t^1, \ldots, f_t^{k-2}\right)'.$$

The resulting linear system can be written in the form

$$A_k \mathbb{E}_t z_{t+1} = B_k z_t + C_k e_t, \qquad e_t = e_t^i. \tag{12}$$

Appendix C reports the corresponding matrices. The system dimension increases linearly with the target horizon, reflecting the additional forward-looking auxiliary variables introduced by the policy rule. Since the system is now $k \times k$, the determinacy conditions used in the $k = 1, 2$ case no longer apply. To assess whether monetary policy can achieve determi-

nacy alone, we analyze the eigenvalues of the transition matrix $A_k^{-1}B_k$. Let $\beta \in (0,1)$, $\kappa > 0$, and $\phi_\pi > 0$. The characteristic polynomial associated with the transition matrix is given by

$$P_k(\lambda) = \lambda^k + \frac{\beta}{\phi_\pi \kappa}\lambda^2 - \frac{\beta + \kappa + 1}{\phi_\pi \kappa}\lambda + \frac{1}{\phi_\pi \kappa}. \tag{13}$$

A notable feature of the characteristic polynomial (13) is that all coefficients on $\lambda^{k-1}, \ldots, \lambda^3$ vanish. Thus, extending the target horizon increases the polynomial degree only through the leading term λ^k, while leaving all lower-order coefficients unchanged.

Analytical solution for $k = 3$. To characterize the determinacy boundary analytically, we first consider the special case $k = 3$, for which the unit-circle conditions admit a closed-form solution. The relevant matrices are again given in Appendix C. The characteristic polynomial associated with $A_3^{-1}B_3$ is given by

$$P_3(\lambda) = \lambda^3 + \frac{\beta}{\phi_\pi \kappa}\lambda^2 - \frac{\beta + \kappa + 1}{\phi_\pi \kappa}\lambda + \frac{1}{\phi_\pi \kappa}. \tag{14}$$

Equilibrium determinacy requires all three eigenvalues of the transition matrix to satisfy $|\lambda| > 1$. Consequently, the boundary between determinacy and indeterminacy is reached when at least one Eigenvalue lies exactly on the unit circle. We therefore study the unit-circle crossing

$$\lambda = e^{i\theta}, \quad \theta \in [0, \pi].^7 \tag{15}$$

which includes the special cases $\lambda = 1 \longleftrightarrow \theta = 0$ and $\lambda = -1 \longleftrightarrow \theta = \pi$. Here, $e^{i\theta}$ denotes Euler's formula, θ indexes the frequency of a potential self-sustaining cycle, and i denotes the imaginary unit (see, for example, Moskowitz, 2002). Starting from the characteristic equation

$$\phi_\pi \kappa \lambda^3 + \beta\lambda^2 - (\beta + \kappa + 1)\lambda + 1 = 0, \tag{16}$$

divide by λ and impose $\lambda = e^{i\theta}$ to obtain

$$\phi_\pi \kappa e^{i2\theta} + \beta e^{i\theta} - (\beta + \kappa + 1) + e^{-i\theta} = 0. \tag{17}$$

Using Euler's formula,

$$e^{i\theta} = \cos\theta + i\sin\theta \quad e^{i2\theta} = \cos 2\theta + i\sin 2\theta \quad e^{-i\theta} = \cos\theta - i\sin\theta,$$

[7] Since the characteristic polynomial has real coefficients, non-real roots occur in complex-conjugate pairs. Hence, restricting attention to $\theta \in [0, \pi]$ is without loss of generality, as frequencies $\theta \in (\pi, 2\pi)$ correspond to the complex conjugates of roots associated with $2\pi - \theta \in (0, \pi)$.

and since a complex number is zero if and only if both its real and imaginary components are zero, the following two equations must hold:

$$\phi_\pi \kappa \cos 2\theta + (\beta + 1)\cos\theta - (\beta + \kappa + 1) = 0, \tag{18}$$

$$\phi_\pi \kappa \sin 2\theta + (\beta - 1)\sin\theta = 0. \tag{19}$$

Three types of unit-circle crossings can occur: a crossing at $\lambda = 1$, a crossing at $\lambda = -1$, and a crossing of a complex-conjugate pair corresponding to $\theta \in (0, \pi)$. Case 1 corresponds to the first indeterminacy possibility discussed in Section 3. Cases 2 and 3 fall in the second indeterminacy possibility. We consider each case in turn.

Case 1: $\theta = 0$ ($\lambda = 1$). Substituting this into (18) yields:

$$\phi_\pi = 1, \tag{20}$$

while condition (19) is trivially fulfilled, as $\sin 0 = 0$. This corresponds to the Taylor principle boundary.

Case 2: $\theta = \pi$ ($\lambda = -1$). Substituting into (18) gives

$$\phi_\pi = 1 + 2\frac{1+\beta}{\kappa}, \tag{21}$$

with (19) being fulfilled. This corresponds to the upper bound on ϕ_π in equation (6). In the general case below, we show that this restriction is not binding for $k > 2$.

Case 3: $\theta \in (0, \pi)$. In this case, $\sin\theta \neq 0$, so the crossing involves a complex-conjugate pair of roots and condition (19) is no longer automatically fulfilled. Instead, using $\sin 2\theta = 2\sin\theta\cos\theta$, the imaginary part (19) implies

$$\cos\theta = \frac{1-\beta}{2\phi_\pi \kappa}. \tag{22}$$

Substituting this expression into (18), observing $\cos 2\theta = 2(\cos\theta)^2 - 1$, and simplifying yields

$$1 - \beta - \phi_\pi \kappa(\beta + \kappa + 1) - (\phi_\pi \kappa)^2 = 0. \tag{23}$$

Equation (23) characterizes the boundary at which a complex-conjugate pair of roots lies on the unit circle. Solving for ϕ_π under the restrictions $\kappa > 0$ and $\phi_\pi > 0$ yields the upper bound $\bar{\phi}_\pi$:

$$\bar{\phi}_\pi(3, \beta, \kappa) = \frac{-(\beta + \kappa + 1) + \sqrt{(\beta + \kappa + 1)^2 + 4(1-\beta)}}{2\kappa}. \tag{24}$$

Under the benchmark calibration, (24) yields

$$\bar{\phi}_\pi \approx 0.048 < 1.$$

To determine on which side of the boundaries in Cases 1 and 3 the roots lie outside the unit circle, we apply Jury (1962)'s stability test to the reciprocal polynomial in Appendix D. The resulting conditions imply that determinacy requires

$$\phi_\pi > 1 \qquad \text{and} \qquad \phi_\pi < \bar{\phi}_\pi(3, \beta, \kappa). \tag{25}$$

Hence, a determinacy region can exist only if

$$\bar{\phi}_\pi(3, \beta, \kappa) > 1.$$

Under the benchmark calibration, however, the upper bound associated with the complex-conjugate pair lies strictly below the Taylor-principle lower bound. The two restrictions in (25) therefore cannot be satisfied simultaneously, and the characteristic polynomial $P_3(\lambda)$ cannot have all three roots outside the unit circle. The existence of a determinacy region nevertheless depends critically on the parameterization, particularly on the slope of the Phillips curve. For sufficiently small κ, the ordering of the two boundaries can reverse, so that $\bar{\phi}_\pi(3, \beta, \kappa) > 1$. In this case, the interval

$$1 < \phi_\pi < \bar{\phi}_\pi(3, \beta, \kappa)$$

is non-empty, and all three eigenvalues may lie outside the unit circle. We discuss this relation in Section 5.4.

It is useful to characterize when the monetary-policy block generates two unstable eigenvalues, as this case becomes relevant once fiscal policy is introduced. At the Taylor-principle boundary, $\phi_\pi = 1$, a single real root crosses the unit circle. By contrast, the upper bound $\bar{\phi}_\pi$ is associated with the crossing of a complex-conjugate pair.[8] Hence, for

$$\phi_\pi < \bar{\phi}_\pi < 1,$$

[8]For a cubic polynomial (14), with $a_1 \equiv \beta/\phi_\pi\kappa, a_2 \equiv (\beta + \kappa + 1)/\phi_\pi\kappa, a_3 \equiv 1/\phi_\pi\kappa$, the number of roots inside and outside the unit circle can also be characterized using the conditions reported in Woodford (2001, pp. 672–673). In the present model, the condition $1 + a_1 + a_2 + a_3 < 0$, which is required for the existence of two unstable roots, is equivalent to $\phi_\pi < 1$. Our root-crossing argument provides an equivalent, arguably more intuitive characterization of the same result, which also generalizes to $k > 3$.

the monetary policy block generates two unstable roots.[9] A fiscal block that contributes one additional unstable root without introducing an additional jump variable can then complete the Blanchard–Kahn count. Thus, the root configuration relevant for fiscal stabilization arises under passive monetary policy in the conventional sense ($\phi_\pi < 1$).

General case: $k > 3$. For general k, the upper bound $\bar{\phi}_\pi$ can be derived with the same unit root crossing analysis as for $k = 3$. The relevant restriction arises from the case $\theta \in (0, \pi)$, in which a complex conjugate pair of roots crosses the unit circle. Substituting (15) into the characteristic polynomial (13), dividing by λ and imposing $P_k(e^{i\theta}, \kappa) = 0$, yields:

$$\phi_\pi \kappa e^{i(k-1)\theta} + \beta e^{i\theta} - (\beta + \kappa + 1) + e^{-i\theta} = 0. \tag{26}$$

As above, separating real and imaginary parts gives:

$$\phi_\pi \kappa \cos([k-1]\theta) + (\beta + 1)\cos\theta - (\beta + \kappa + 1) = 0, \tag{27}$$

$$\phi_\pi \kappa \sin\big([k-1]\theta\big) + (\beta - 1)\sin\theta = 0. \tag{28}$$

The two conditions for $\sin\theta = 0$ from above (cases 1 and 2) remain unchanged relative to $k = 3$. Note, however, that the second case of $\theta = \pi$ is generally not binding. To see this, square and add both equations above, which gives

$$(\phi_\pi \kappa)^2 = [1 + \beta + \kappa - (\beta + 1)\cos\theta]^2 + (1 - \beta)^2(1 - \cos^2\theta).$$

Comparing the second case, $\theta = \pi$, and the third case of an interior root for $\theta \in (0, \pi)$ yields

$$(\phi_\pi^{\theta=\pi} \kappa)^2 - (\phi_\pi \kappa)^2 = [2(1 + \beta) + \kappa]^2 - [1 + \beta + \kappa - (\beta + 1)\cos\theta]^2 - (1 - \beta)^2(1 - \cos^2\theta),$$

which is strictly positive for $\theta \in (0, \pi)$, such that the second case of a root at $\theta = \pi$ is non-binding, if existent.[10] In the third case where $\sin\theta \neq 0$, equation (28) implies

$$\phi_\pi \kappa \frac{\sin([k-1]\theta)}{\sin\theta} = 1 - \beta. \tag{29}$$

[9] The same is true for the 'traditional' passive monetary policy $\phi_\pi < 1 < \bar{\phi}_\pi$, but then monetary policy can achieve determinacy alone.

[10] For even values of k, the value of ϕ_π would be negative for the root at $\theta = \pi$, which is ruled out by the condition obtained in case 1, i.e., the Taylor principle.

Using the Chebyshev polynomial of the second kind, which reads as

$$\frac{\sin([k-1]\theta)}{\sin\theta} = U_{k-2}(c),$$

equation (29) can be written as

$$\phi_\pi = \frac{1-\beta}{\kappa U_{k-2}(c)}, \tag{30}$$

with $c \equiv \cos\theta$. Equation (30) characterizes the candidate upper bound. However, we cannot choose c freely, since it must simultaneously satisfy the real part of the characteristic equation. Since $\cos([k-1]\theta) = T_{k-1}(c)$, the real part (27), observing (30), becomes

$$\frac{1-\beta}{U_{k-2}(c)} T_{k-1}(c) + (\beta+1)c - (\beta+\kappa+1) = 0.$$

Therefore, the upper bound of ϕ_π can implicitly be represented as:

$$\bar{\phi_\pi}(k,\beta,\kappa) = \min_{c\in\mathcal{C}} \frac{1-\beta}{\kappa U_{k-2}(c)}$$

with

$$\mathcal{C} = \left\{ c \in (-1,1) : \frac{1-\beta}{U_{k-2}(c)} T_{k-1}(c) + (\beta+1)c - (\beta+\kappa+1) = 0 \right\}$$

Thus, the upper bound on the inflation coefficient can be characterized implicitly by the roots of a polynomial involving Chebyshev polynomials of the first and second kind. In general, no closed-form solution exists for arbitrary target horizons. Nevertheless, numerical evaluations under the benchmark calibration show that $\bar{\phi_\pi}(k,\beta,\kappa) < 1$ for all $k > 1$. Hence, the upper bound implied by the complex-pair crossing lies below the lower bound imposed by the Taylor principle. Increasing the horizon adds additional roots through the term λ^k, reflecting the longer expectational chain implied by the policy rule, but these additional *horizon roots* do not alter the policy-induced instability properties of the system relative to lower values of k. Consequently, for target horizon k, the monetary policy block contains k forward-looking variables but generates at most $k-1$ unstable eigenvalues, leaving the system one unstable root short of the Blanchard–Kahn requirements under the benchmark calibration. The indeterminacy result established for $k = 3$ therefore extends to all longer inflation-targeting horizons. Figure 1 shows the boundaries for alternative values of κ, whose influence is discussed next.

5.4 The Slope of the Phillips Curve and Determinacy

The previous section established that, kor $k > 1$, equilibrium determinacy requires:

$$1 < \phi_\pi < \bar{\phi}_\pi(k, \beta, \kappa).$$

Under the benchmark calibration, the upper bound lies below unity and the interval is empty. However, this result depends crucially on the slope of the Phillips curve.

For $k = 3$, the complex-root boundary (23) is given by

$$1 - \beta - \phi_\pi \kappa(\beta + \kappa + 1) - (\phi_\pi \kappa)^2 = 0, \tag{31}$$

which defines the upper bound $\bar{\phi}_\pi(k, \beta, \kappa)$, see condition (24). Differentiating (31) implicitly with respect to κ yields

$$\frac{d\bar{\phi}_\pi}{d\kappa} = -\frac{\phi_\pi(\beta + 2\kappa + 1)/\kappa + 2\phi_\pi^2}{\beta + \kappa + 1 + 2\phi_\pi \kappa}.$$

Since $\phi_\pi > 0$, $\kappa > 0$, and $\beta \in (0, 1)$, both the numerator and denominator are positive and

$$\frac{d\bar{\phi}_\pi}{d\kappa} < 0.$$

The upper bound on the inflation coefficient therefore decreases with the slope of the Phillips curve. Equivalently, a flatter Phillips curve enlarges the interval of policy parameters consistent with equilibrium determinacy. Section 3.1 provides the economic intuition. A larger value of κ increases the impact of interest-rate decisions based on expected inflation rates, which might go in the opposite direction as current inflation rates. Consequently, the complex-conjugate pair reaches the unit circle only for larger values of ϕ_π, shifting the upper bound upward and enlarging the determinacy region.

Although no closed-form expression exists for arbitrary target horizons, the same mechanism carries over to general k. From equation (30)

$$\phi_\pi = \frac{1 - \beta}{\kappa U_{k-2}(c)}, \tag{32}$$

it follows that, for a given crossing angle c, the candidate upper bound is inversely related to κ. Figure 1 illustrates this result. For sufficiently steep Phillips curves, such as the benchmark calibration $\kappa = 0.1$, the upper bound remains below the Taylor-principle threshold for all $k > 1$, so that no determinacy region exists. As the Phillips curve flattens, however, the upper bound shifts upward and eventually exceeds unity. In that case, the interval $1 < \phi_\pi < \bar{\phi}_\pi(k, \beta, \kappa)$ becomes non-empty and monetary policy is again capable of generating enough unstable eigenvalues to satisfy the Blanchard–Kahn conditions.

These results suggest that the conventional classification of policy regimes in the sense of Leeper (1991) becomes incomplete under inflation forecast targeting. In the standard framework, an active monetary policy regime is characterized solely by the Taylor principle. Under forward-looking inflation targeting, however, determinacy depends jointly on the aggressiveness of monetary policy and the structural transmission mechanism embodied in the Phillips curve. A sufficiently flat Phillips curve enlarges the set of policy parameters consistent with determinacy, whereas a steep Phillips curve may eliminate this set entirely.

5.5 Welfare Implications of Inflation Forecast Targeting

The previous sections characterized the conditions under which forward-looking Taylor rules deliver equilibrium determinacy. These results are positive in nature and do not address whether the resulting equilibrium is desirable from a welfare perspective. To answer this question, we derive the minimum-state-variable (MSV) solution and determine the welfare-optimal inflation-response coefficient for different forecast horizons. To this end, we consider the standard quadratic loss function, see Woodford (2003),

$$\mathcal{L}_0 = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \zeta y_t^2\right), \tag{33}$$

where ζ denotes the relative weight on output-gap stabilization. Under the standard micro-founded approximation to household welfare, it is proportional to the slope of the Phillips curve. In particular, following Galí (2008), one obtains:

$$\zeta = \frac{\kappa}{\epsilon}$$

where $\epsilon > 1$ denotes the elasticity of substitution across differentiated goods.[11] To obtain a meaningful inflation-output trade-off, we introduce a cost-push shock,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t, \qquad u_t = \rho_u u_{t-1} + \varepsilon_t^u. \tag{34}$$

Without a cost-push disturbance, divine coincidence aligns inflation and output stabilization, and the welfare problem becomes trivial. The MSV solution takes the form[12]

$$\pi_t = a_\pi(k, \phi_\pi) u_t \qquad y_t = a_y(k, \phi_\pi) u_t,$$

[11] More generally, the welfare weight is proportional to κ, so the subsequent argument does not depend on the exact normalization.

[12] Detailed derivations can be found in Appendix E.

Solving for a_π and a_y yields:

$$a_y = \frac{\rho_u - \phi_\pi \rho_u^k}{1 - \rho_u} a_\pi \qquad a_\pi = \frac{\kappa a_y + 1}{1 - \beta \rho_u}.$$

Hence, welfare depends, besides ρ_u only on

$$\bar{\mathcal{L}}(k, \phi_\pi) = a_\pi^2 + \zeta a_y^2. \tag{35}$$

Defining

$$m(k, \phi_\pi) \equiv \frac{\rho_u - \phi_\pi \rho_u^k}{1 - \rho_u},$$

it follows that

$$a_y = m a_\pi,$$

and minimizing the loss function with respect to ϕ_π yields

$$m^* = -\frac{\kappa}{\zeta(1 - \beta \rho_u)} = -\frac{\epsilon}{1 - \beta \rho_u}.$$

Substituting back gives the unconstrained MSV-optimal inflation coefficient

$$\phi_\pi^*(k) = \frac{\rho_u + \epsilon(1 - \rho_u)/(1 - \beta \rho_u)}{\rho_u^k}. \tag{36}$$

Since $\rho_u < 1$, the denominator ρ_u^k decreases with the forecast horizon. Consequently, the welfare-optimal inflation coefficient increases with k. Longer forecast horizons therefore require increasingly aggressive responses to expected future inflation.

The previous sections have shown that equilibrium determinacy imposes an upper bound $\phi_\pi < \bar{\phi}_\pi(k, \beta, \kappa)$, which declines with increasing k. By contrast, equation (36) implies that the welfare-optimal inflation response increases with the target horizon. Hence, as the target horizon increases, the welfare-optimal inflation coefficient and the size of the determinacy region move in opposite directions. Figure 2 illustrates this interaction. For sufficiently short target horizons, the welfare-optimal coefficient lies inside the determinacy region and can therefore be implemented as a unique rational expectations equilibrium. Beyond a critical horizon $\bar{k}$, however, the welfare-optimal coefficient exceeds the largest value compatible with equilibrium determinacy. Consequently, the welfare-optimal rule is no longer implementable as a unique equilibrium. The critical horizon $\bar{k}$ depends on the slope of the New Keynesian Phillips curve. A flatter Phillips curve enlarges the determinacy region and therefore increases the largest forecast horizon for which the welfare-maximizing Taylor coefficient remains im-

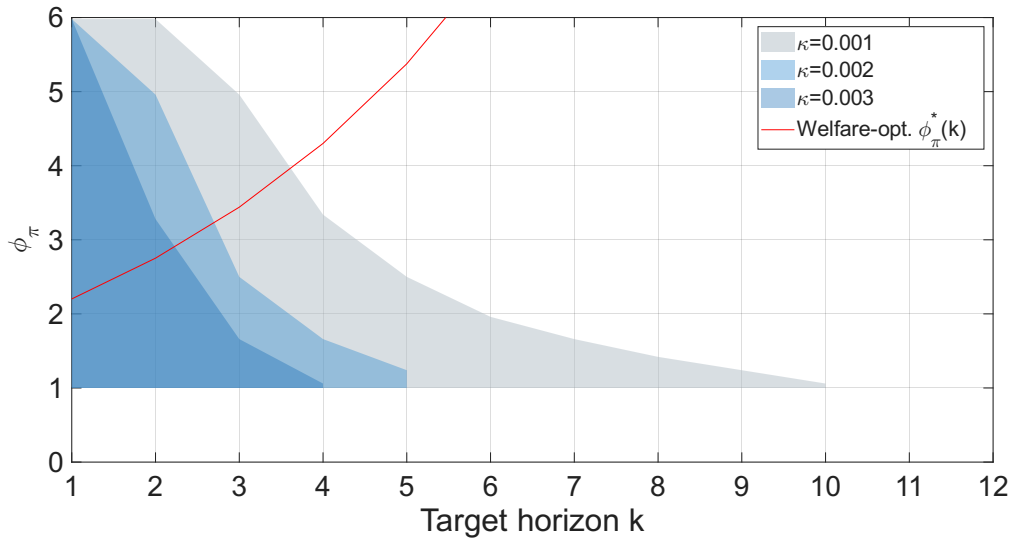


Figure 2: Determinacy region as a function of target horizon k and policy parameter ϕ_π for different values of κ with $\beta = 0.99$ and the welfare maximizing $\phi_\pi^*(k)$.

plementable. Nevertheless, because the welfare-optimal inflation response rises exponentially with the forecast horizon while the determinacy region shrinks, sufficiently distant inflation targets eventually generate a conflict between welfare optimization and equilibrium uniqueness.

These results suggest that central banks should exercise caution when targeting inflation forecasts at increasingly distant horizons. Although a flatter Phillips curve allows for active monetary policy and enlarges the set of implementable policy rules, sufficiently forward-looking inflation targeting may force policymakers to choose between welfare optimization and equilibrium determinacy.

6 Fiscal Policy and restored determinacy

We now introduce fiscal policy and show that it can, theoretically, restore equilibrium determinacy for any target horizon k. As derived above, real government debt and the primary surplus evolve according to (4) and (5). Substituting the surplus rule (5) into the government budget constraint (4), and using the generalized forward-looking Taylor rule (9) as well as the definitions in (10), yields the law of motion for real government debt:

$$v_t = \frac{1}{\beta}(1-\alpha_b)v_{t-1} + \frac{\gamma\phi_\pi}{\beta^{k-1}}\mathbb{E}_t\pi_{t+1} - \frac{\gamma}{\beta}\pi_t - \frac{\alpha_y}{\beta}y_t$$
$$- \gamma\phi_\pi\kappa\sum_{j=1}^{k-2}\frac{1}{\beta^{k-j}}\mathbb{E}_t f_{t+1}^j + \frac{1}{\beta}(\alpha_b - 1)e_t^s + \gamma e_t^i. \tag{37}$$

Equation (37) highlights the key valuation channels through which monetary policy affects fiscal dynamics: inflation and the nominal interest rate, here substituted by the Taylor rule, enter the government budget constraint directly, while output affects debt through the surplus rule.

Augmented system. Adding the fiscal block to the monetary policy block expands the state vector to

$$z_t = \left(v_{t-1}, \pi_t, y_t, f_t^1, \ldots, f_t^{k-2}\right)'.$$

Importantly, lagged real government debt v_{t-1} is predetermined and therefore does not constitute a jump variable. The resulting linear system can again be written as

$$A_{k+1}\mathbb{E}_t z_{t+1} = B_{k+1} z_t + C_{k+1} e_t, \qquad e_t = (e_t^i, e_t^s)'. \tag{38}$$

Appendix F reports the corresponding matrices. The first row of each matrix corresponds to the fiscal block, while the remaining rows correspond to the monetary policy block analyzed in the previous section.

Restored determinacy. The transmission matrix $A_{k+1}^{-1}B_{k+1}$ inherits the system's block structure. In particular, the first column of $A_{k+1}^{-1}B_{k+1}$ is given by

$$\left(\frac{1-\alpha_b}{\beta}, 0, \ldots, 0\right)',$$

so that $(1-\alpha_b)/\beta$ is an eigenvalue of the system. This eigenvalue reflects fiscal policy's response to lagged government debt and is independent of the target horizon k. For this fiscal eigenvalue to lie outside the unit circle, it must satisfy

$$\frac{1-\alpha_b}{\beta} > 1, \qquad \text{or equivalently} \qquad \alpha_b < 1-\beta. \tag{39}$$

The condition $\alpha_b < 1-\beta$ implies that fiscal policy is active in the sense that primary surpluses do not stabilize government debt. Under this condition, the fiscal block contributes an additional unstable eigenvalue through the government budget constraint. Since lagged government debt is predetermined, this eigenvalue is added without introducing an additional jump variable. For all $k > 1$, the monetary policy block alone falls short of the Blanchard–Kahn conditions by exactly one unstable root under the benchmark calibration of the Phillips-curve slope ($\kappa = 0.1$). More generally, this is the case whenever $\bar{\phi}_\pi(k, \beta, \kappa) < 1$, see Section 5. In that region, monetary policy alone cannot generate enough unstable eigenvalues to pin

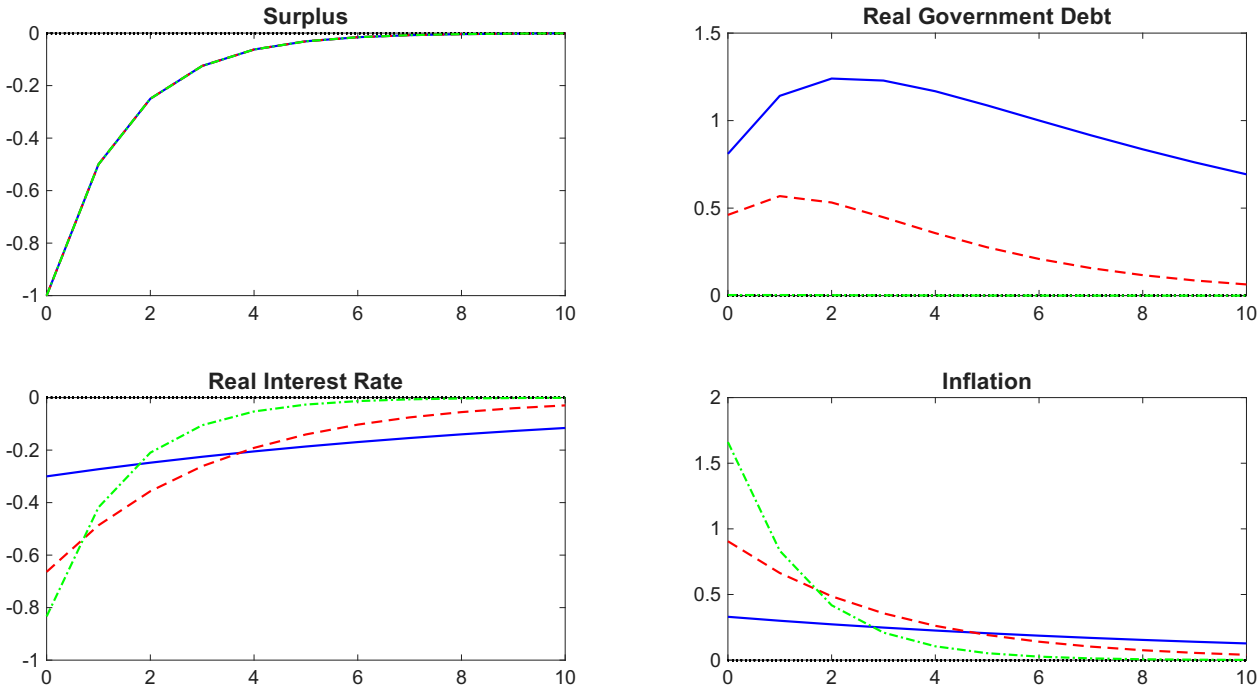


Figure 3: Impulse response functions after negative surplus shock of unity for $k = 3$, $\beta = 0.99, \phi_\pi = 0, \alpha_b = 0, \alpha_y = 0, \rho_g = 0.5, \gamma = 0.6$, with $\kappa = 0.1$ (blue, solid lines), $\kappa = 0.5$ (red, dashed lines), $\kappa = 0.01$ (green, dash-dotted lines).

down expectations. Crucially, because the fiscal eigenvalue $(1 - \alpha_b)/\beta$ is independent of the inflation-targeting horizon k, the fiscal block supplies precisely this missing root, thereby closing the root–jump mismatch. Remember that this requires the central bank to set ϕ_π below $\bar{\phi}_\pi(k, \beta, \kappa)$, as otherwise more than one root would become stable. As a result, the combined monetary–fiscal system admits a unique rational expectations equilibrium even for arbitrarily forward-looking Taylor rules. The corresponding intuition is given in Section 3.2.

Figure 3 shows impulse response functions in such a fiscal dominant regime for $k = 3$ and different values of κ. For illustration, we choose an extreme calibration, under which monetary policy is completely passive, i.e., the central bank does not react to inflation at all, $\phi_\pi = 0$, implying a constant nominal interest rate. Fiscal policy, in contrast, is fully active. That is, it does not react to the debt level, nor to the output gap: $\alpha_b = \alpha_y = 0$. We also assume a steady-state debt-to-GDP ratio of $\gamma = 0.6$. The remaining parameters are given below the figure. The impulse is a negative surplus shock of $e_0^s = -1$. The negative surplus shock increases real government debt. Since fiscal policy does not subsequently raise the surplus above its steady-state value, inflation must rise to inflate away government debt over time, as reflected in the negative response of the real interest rate. With a fairly high $\kappa = 0.5$, inflation reacts strongly to the fall in the real interest rate, so real government debt does not rise throughout.

By contrast, in a monetary-led regime with an active Taylor rule, the central bank would raise the nominal interest rate in response to higher inflation, thereby stabilizing inflation and preventing debt from returning to its pre-shock value. Hence, fiscal adjustments rather than price-level movements would need to ensure government solvency.

7 Additional anchors and alternative policy rules

We now examine whether the indeterminacy associated with inflation forecast targeting is robust to alternative monetary transmission mechanisms and policy rules. We consider four extensions: transmission lags of monetary policy action, contemporaneous output-gap feedback, interest rate smoothing, and average inflation forecast targeting. While these modifications differ in their economic interpretation, they share a common implication: they weaken the destabilizing effect of inflation forecast targeting and enlarge the determinacy region. Importantly, however, they do not eliminate the underlying horizon effect. As the policy horizon increases, progressively stronger stabilizing forces are required to preserve determinacy.

Transmission lags. Thus far, the baseline model abstracts from transmission lags and therefore implies that inflation responds immediately to monetary policy. Since one of the main motivations for inflation forecast targeting is the presumption that monetary policy affects inflation only gradually, we now examine whether our determinacy results continue to hold once delayed inflation adjustment is incorporated. To capture this feature, we replace the purely forward-looking New Keynesian Phillips curve with its hybrid counterpart (Galí and Gertler, 1999),

$$\pi_t = (1-\rho_\pi)\beta\mathbb{E}_t\pi_{t+1} + \kappa y_t + \rho_\pi\pi_{t-1}, \tag{40}$$

where $\rho_\pi \in [0,1)$ determines the importance of the backward-looking component in inflation dynamics. As inflation becomes partially backward-looking, monetary policy is transmitted to inflation more gradually.

Figure 4 illustrates this mechanism. It reports the impulse responses to a contractionary monetary policy shock of $e_t^i = -1$ with an autocorrelation of 0.75 for various degrees of transmission lags.[13] Unlike in the baseline model, inflation now responds gradually and exhibits a hump-shaped adjustment, implying that the peak effect of monetary policy on inflation occurs only after several periods. This effect is stronger for larger values of ρ_π. To investigate whether introducing delayed inflation adjustment changes the determinacy properties of inflation forecast targeting, we conduct the same analysis as in our benchmark

[13]The shock follows an AR(1) process to generate more realistic impulse responses. This persistence does not affect the model's determinacy properties.

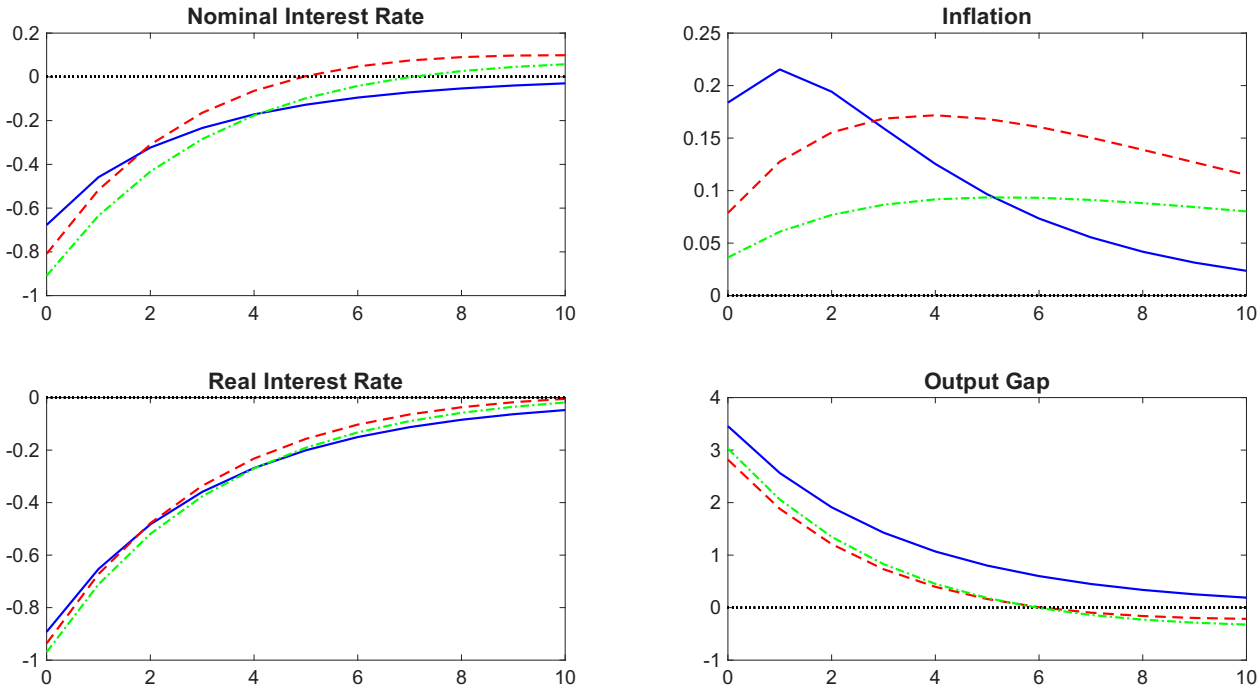


Figure 4: Impulse response functions after contractionary monetary policy shock of unity for $k=1, \beta=0.99$, $\kappa=0.1, \phi_\pi=1.5, \rho_\nu=0.75, \gamma=0$, with $\rho_\pi=0.3$ (blue, solid lines), $\rho_\pi=0.6$ (red, dashed lines), $\rho_\pi=0.9$ (green, dash-dotted lines).

model. The monetary policy block can be represented by the state vector

$$z_t = (\pi_t, y_t, f_t^1, \ldots, f_t^{k-2}, \pi_{t-1})'.$$

The associated transition matrix of the matrix representation of the model implies the characteristic polynomial

$$P_k(\lambda, \rho_\pi) = \lambda^{k+1} + \frac{\beta(1-\rho_\pi)}{\phi_\pi \kappa}\lambda^3 - \frac{\beta(1-\rho_\pi)+\kappa+1}{\phi_\pi \kappa}\lambda^2 + \frac{1+\rho_\pi}{\phi_\pi \kappa}\lambda - \frac{\rho_\pi}{\phi_\pi \kappa}. \tag{41}$$

As in the baseline model, extending the target horizon affects the characteristic polynomial only through the leading term λ^{k+1}. Inflation inertia therefore does not alter the source of instability associated with longer inflation forecast horizons. Instead, it modifies the lower-order coefficients of the polynomial via the parameter ρ_π, thereby shifting the eigenvalues. Consequently, sufficiently strong inflation persistence may restore determinacy for moderate forecast horizons. We characterize the determinacy boundary using the same unit-circle argument as in the baseline model. Appendix G provides a detailed derivation. As shown in Figure 5, under the baseline calibration, the threshold $\rho^{min}(k)$ above which we reach determinacy increases monotonically with k. While inflation inertia restores determinacy

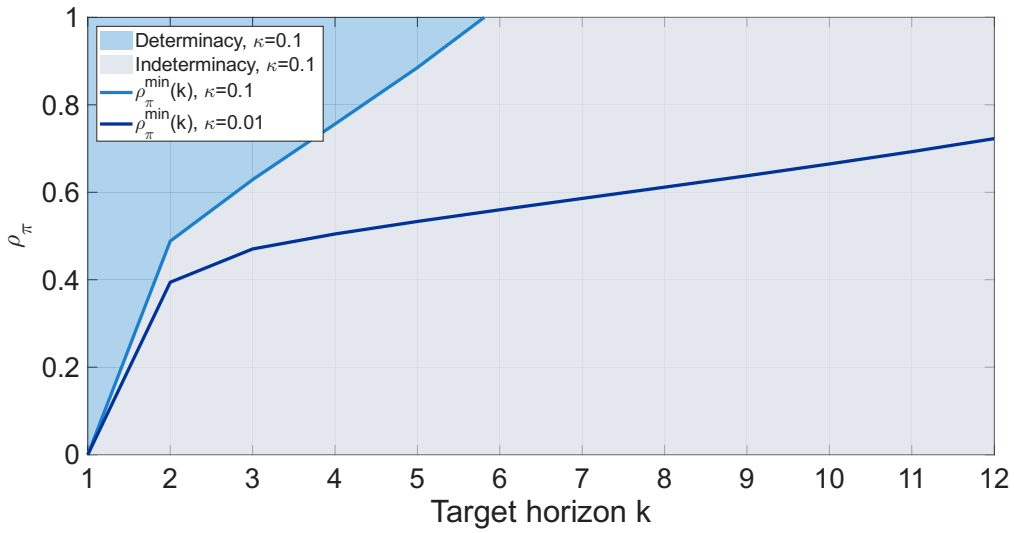


Figure 5: Determinacy as a function of target horizon k and inflation inertia ρ_π for $\beta = 0.99, \phi_\pi = 1.5$.

for horizons up to $k = 5$, the required value exceeds unity for $k \geq 6$ when $\kappa = 0.1$. Since $\rho_\pi \in [0, 1)$, intrinsic inflation persistence alone cannot stabilize the economy once the target horizon becomes sufficiently long. By contrast, when $\kappa = 0.01$, the threshold remains below one even for larger target horizons, implying that inflation inertia can sustain determinacy for significantly longer forecast horizons.

Hence, introducing inflation inertia weakens, but does not overturn, the baseline determinacy results. By slowing the adjustment of inflation, delayed inflation responses increase persistence in the economy and tighten the link between current inflation, which the central bank can control most effectively, and inflation at the target horizon. This enlarges the region of determinacy. However, the stabilizing effect is limited. As the target horizon increases, the number of forward-looking variables continues to rise, while inflation inertia merely modifies the adjustment of inflation rather than eliminating the additional forward-looking dynamics. Consequently, sufficiently long forecast horizons still lead to indeterminacy under empirically plausible degrees of inflation persistence.

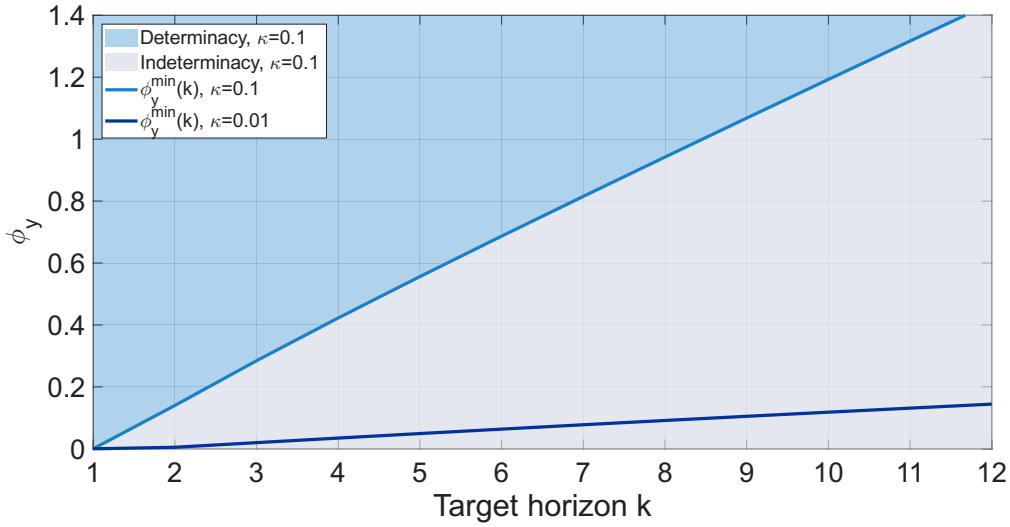


Figure 6: Determinacy region as a function of target horizon k and output-gap coefficient ϕ_y for $\beta = 0.99, \phi_\pi = 1.5$.

Output-gap stabilization. We next allow the policy rate to respond contemporaneously to the output gap. The Taylor rule becomes

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+k} + \phi_y y_t + e_t^i. \tag{42}$$

Again, we investigate the resulting changes in the determinacy properties by deriving the characteristic polynomial of the associated transmission matrix:

$$P_k(\lambda, \phi_y) = \lambda^k + \frac{\beta}{\phi_\pi \kappa}\lambda^2 - \frac{\beta(1+\phi_y) + \kappa + 1}{\phi_\pi \kappa}\lambda + \frac{1+\phi_y}{\phi_\pi \kappa}. \tag{43}$$

As above, we use this to derive the determinacy threshold for ϕ_y. Figure 6 illustrates the resulting determinacy regions as a function of k and ϕ_y for $\kappa = 0.1$ and $\kappa = 0.01$.[14] Allowing the Taylor rule to respond to the output gap introduces a contemporaneous feedback channel into monetary policy. Unlike inflation forecasts on long horizons, the output gap directly affects current inflation through the New Keynesian Phillips curve. As a result, the Taylor rule becomes, in a sense, less forward-looking by linking the interest-rate decision to current developments. This restores the possibility of equilibrium determinacy for longer inflation-targeting horizons, but only if it is sufficiently strong. As the target horizon k increases, the policy rule becomes more forward-looking and introduces additional expectational degrees of freedom. To offset this effect, the response to the output gap must rise. Accordingly, the minimum value of ϕ_y required for determinacy increases monotonically with the inflation-targeting horizon.

[14]The derivation is provided in Appendix H in the Online Appendix.

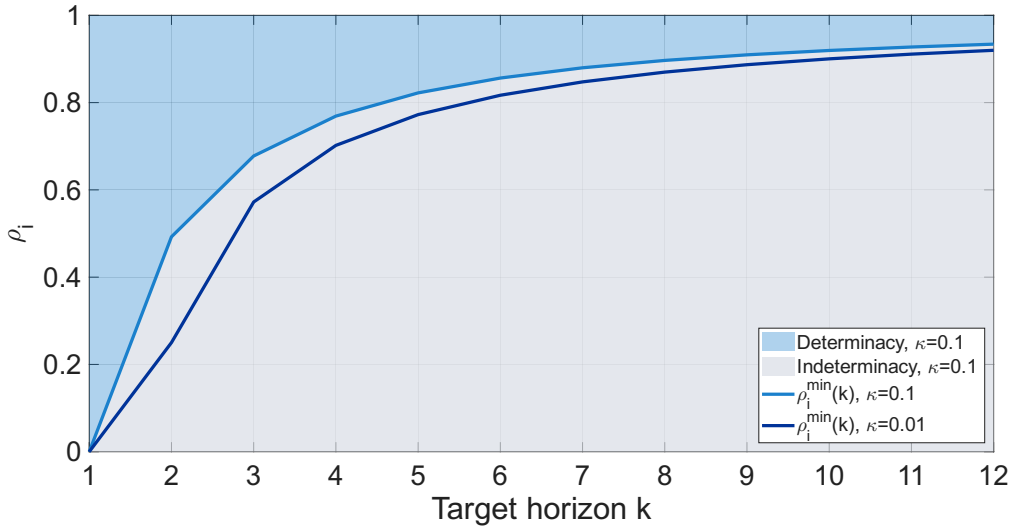


Figure 7: Determinacy region as a function of target horizon k and interest rate smoothing ρ_i for $\beta = 0.99, \phi_\pi = 1.5$.

Interest rate smoothing. Third, we introduce interest rate smoothing into the Taylor rule, allowing the policy rate to depend on its own lagged value. The policy rule becomes

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \mathbb{E}_t \pi_{t+k} + e_t^i, \tag{44}$$

where $\rho_i \in [0, 1)$ captures the degree of policy inertia. The monetary policy block can be represented with the state vector

$$z_t = \left(\pi_t, y_t, f_t^1, \ldots, f_t^{k-2}, i_{t-1}\right)'.$$

The associated transition matrix implies the characteristic polynomial

$$P_k(\lambda, \rho_i) = (1-\rho_i)\lambda^{k+1} + \frac{\beta}{\phi_\pi \kappa}\lambda^3 - \frac{\beta(1+\rho_i)+\kappa+1}{\phi_\pi \kappa}\lambda^2 + \frac{1+\rho_i(\beta+\kappa+1)}{\phi_\pi \kappa}\lambda - \frac{\rho_i}{\phi_\pi \kappa}. \tag{45}$$

Applying the same boundary analysis as above yields a minimum degree of interest rate smoothing required for determinacy. Figure 7 illustrates the determinacy region as a function of the target horizon k and the smoothing parameter ρ_i, again for $\kappa = 0.1$ and $\kappa = 0.01$ (the derivation is provided in Online Appendix I). Interest rate smoothing allows monetary policy to regain control over expectations by introducing inertia in the policy rate. The interest rate at the target horizon is more closely tied to current policy decisions that respond to expected inflation at that horizon. This inertia acts as a nominal anchor and can restore equilibrium determinacy even when the Taylor rule targets inflation at long horizons. However, the required smoothing is substantial and increases with the target horizon k, reflecting the need for policy inertia to offset the growing forward-lookingness of monetary policy.

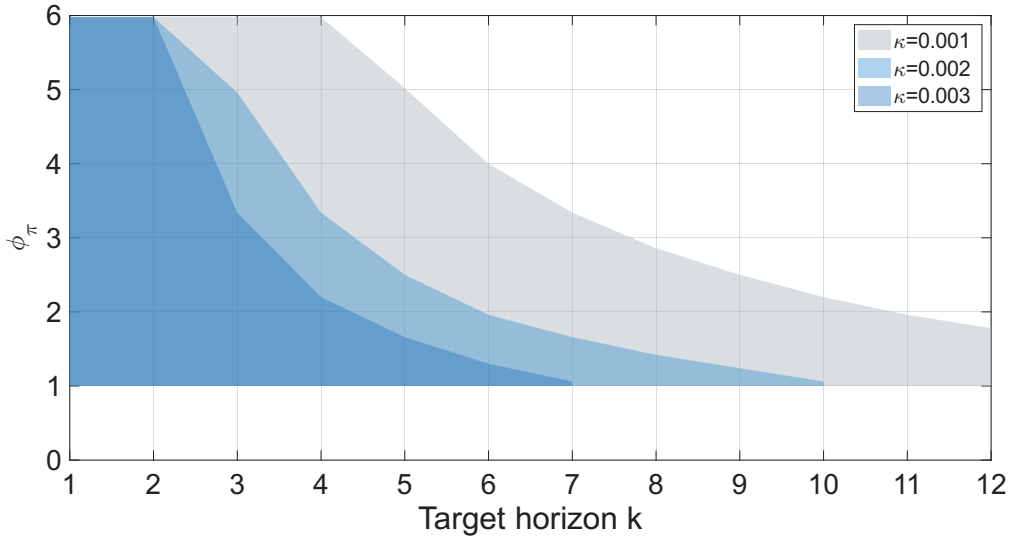


Figure 8: Determinacy region as a function of target horizon k and policy parameter ϕ_π for different values of κ with $\beta = 0.99$.

Average Inflation Forecast Targeting. So far, monetary policy has been assumed to respond exclusively to inflation at a single forecast horizon $t + k$, motivated by the ECB's approach to 'look through' short-term inflation movements. We now consider an alternative specification in which the central bank reacts to average expected inflation over the forecast window (Clarida et al., 1998, 2000). To preserve the purely forward-looking nature of monetary policy, we exclude contemporaneous inflation and specify

$$i_t = \frac{\phi_\pi}{k} \sum_{j=1}^{k} \mathbb{E}_t \pi_{t+j} + e_t^i. \tag{46}$$

The point-targeting rule in the baseline model places all policy weight on inflation at the terminal horizon $t+k$. By contrast, equation (46) distributes the same total weight uniformly across all forecast horizons from $t+1$ to $t+k$. The characteristic polynomial associated with equation (46) is

$$P_k(\lambda) = \frac{\phi_\pi \kappa}{k} \sum_{j=1}^{k} \lambda^j + \beta\lambda^2 - (\beta + \kappa + 1)\lambda + 1. \tag{47}$$

Relative to the baseline rule, average inflation forecast targeting replaces the terminal-horizon term $\phi_\pi \kappa \lambda^k$ with

$$\frac{\phi_\pi \kappa}{k} \left(\lambda + \lambda^2 + \cdots + \lambda^k\right).$$

The response to the terminal inflation forecast is therefore diluted, while shorter-horizon inflation forecasts also enter the policy rule. This weakens the destabilizing effect of target-

ing inflation far into the future, as monetary policy reacts more directly to current (non-fundamental) inflation movements. Figure 8 shows the resulting determinacy regions for different values of κ. Relative to point inflation forecast targeting, the determinacy region is larger. Nevertheless, the baseline model's qualitative results remain unchanged. For a given κ, the determinacy region shrinks as the forecast window becomes longer. Moreover, a steeper Phillips curve reduces the set of monetary-policy coefficients consistent with determinacy. Hence, averaging inflation forecasts attenuates, but does not eliminate, the indeterminacy mechanism associated with forward-looking monetary policy.

Taken together, these extensions confirm that the determinacy problem identified in the baseline model is not an artifact of instantaneous inflation adjustment or of the particular specification of the Taylor rule. Inflation inertia, output-gap feedback, interest rate smoothing, and averaging inflation forecasts can all enlarge the set of policy rules consistent with determinacy. Yet none eliminates the underlying horizon effect. As monetary policy becomes increasingly forward-looking, stronger stabilizing forces are required to offset the additional expectational dynamics. The extensions therefore shift the determinacy boundary without overturning the central result: targeting inflation further into the future makes equilibrium uniqueness increasingly difficult to sustain.

7.1 Fiscal Policy

Importantly, our results in this section do not alter the role of fiscal policy established in Section 6. Whenever the additional stabilizing mechanisms considered above are insufficient for monetary policy alone to ensure determinacy, fiscal policy can provide the missing nominal anchor in a fiscally led regime. That is, our results change only if the discussed extensions restore determinacy. In this case, as usual, an active monetary policy with passive fiscal policy, or vice versa, is possible.

8 Conclusion

This paper shows that medium-term inflation forecast targeting confronts central banks with a fundamental trade-off: If the slope of the New Keynesian Phillips curve is sufficiently small, monetary policy can ensure determinacy, but only at the cost of deviating from the welfare-maximizing Taylor coefficient. For even longer target horizons or a flatter Phillips Curve, determinacy requires a fiscally led equilibrium, in which the central bank relinquishes control over the price level. The underlying mechanism stems from the forward-looking nature of monetary policy. When the Taylor rule responds to inflation sufficiently far in the future,

monetary policy alone may fail to deliver a unique equilibrium, as the policy reaction is not reactive enough to short-term developments to discipline current expectations.

A central contribution of this paper is to show how determinacy hinges not only on the lower bound of the Taylor coefficient, but also on an upper bound, $\bar{\phi}_\pi(k, \beta, \kappa)$, which depends on the slope of the New Keynesian Phillips curve. As the Phillips curve becomes flatter (i.e., as κ declines), this upper bound increases, thereby expanding the range of Taylor coefficients consistent with equilibrium determinacy. Conversely, if the Phillips curve is sufficiently steep, or the target horizon sufficiently long, the upper bound falls below the lower bound required by the Taylor principle. In that case, the monetary policy block cannot generate enough unstable eigenvalues to pin down expectations, regardless of how aggressively the central bank responds to expected inflation. Equilibrium determinacy, therefore, depends jointly on the design of the monetary policy rule and the structural characteristics of the economy. We then show that introducing fiscal policy fundamentally changes this conclusion. When the upper bound lies below the lower bound, equilibrium determinacy can be restored by an active fiscal policy that does not stabilize government debt. In this case, the government budget constraint becomes binding in determining the price level, and fiscal policy supplies the missing restriction that monetary policy alone fails to provide. Restored determinacy, therefore, corresponds to an active fiscal and passive monetary policy regime. In this sense, inflation forecast targeting may invite fiscal dominance.

Finally, we show that adding conventional nominal anchors to the Taylor rule, such as a response to the output gap or interest rate smoothing, can partially restore monetary determinacy when these anchors are sufficiently strong. However, under a purely forward-looking Taylor rule, such regime switching remains constrained by the structural environment. Dependent on parameter values, monetary policy alone may remain unable to anchor expectations, making fiscal policy essential for restoring equilibrium determinacy.

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Appendix

A Determinacy with a one-period target horizon (k = 1)

For $k = 1$, the Taylor rule is given by

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1} + e_t^i.$$

Inserting the Taylor rule in the IS equation (1) yields:

$$y_t = \mathbb{E}_t y_{t+1} + (1 - \phi_\pi)\mathbb{E}_t \pi_{t+1} - e_t^i \tag{48}$$

Together with the New Keynesian Phillips curve, this equation characterizes the dynamics of inflation and output under a forward-looking Taylor rule with $k = 1$.

Determinacy. The IS equation and the Phillips curve form a linear system in (π_t, y_t). Writing the system in matrix form as

$$A_1 \mathbb{E}_t z_{t+1} = B_1 z_t + C_1 e_t, \qquad z_t = (\pi_t, y_t)', \tag{49}$$

the matrices A_1 and B_1 are given by

$$A_1 = \begin{pmatrix} 1 - \phi_\pi & 1 \\ \beta & 0 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 0 & 1 \\ 1 & -\kappa \end{pmatrix}. \tag{50}$$

The matrix A_1 is invertible for all admissible parameter values. The reduced-form transition matrix is therefore given by $A_1^{-1}B_1$. The transmission matrix is:

$$A_1^{-1}B_1 = \begin{pmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \phi_\pi - 1 & 1 - \frac{\kappa(\phi_\pi - 1)}{\beta} \end{pmatrix} \tag{51}$$

A unique rational expectations equilibrium exists if the number of eigenvalues of $A_1^{-1}B_1$ outside the unit circle equals the number of jump variables (Blanchard and Kahn, 1980). Since both inflation and output are forward-looking, the system has two jump variables and therefore requires two eigenvalues outside the unit circle. For a 2×2 system, Woodford (2003) shows that this condition is equivalent to

$$\det(A_2^{-1}B_2) > 1, \quad \det(A_2^{-1}B_2) - \mathrm{tr}(A_2^{-1}B_2) > -1, \quad \det(A_2^{-1}B_2) + \mathrm{tr}(A_2^{-1}B_2) > -1.$$

Evaluating these conditions yields

$$1 < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa}. \tag{52}$$

Hence, under a purely forward-looking Taylor rule with $k = 1$, equilibrium determinacy is achievable without fiscal policy, provided that monetary policy satisfies the Taylor principle ($\phi_\pi > 1$) and the additional upper bound derived above. The same logic applies for $k = 0$.

B Indeterminacy with a two-period target horizon (k = 2)

For $k = 2$, the Taylor rule is given by

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+2} + e_t^i.$$

To express the system in first-order form, we use the New Keynesian Phillips curve. Iterating the Phillips curve one period ahead and solving for expected inflation two periods ahead gives

$$\mathbb{E}_t \pi_{t+2} = \frac{1}{\beta}\mathbb{E}_t \pi_{t+1} - \frac{\kappa}{\beta}\mathbb{E}_t y_{t+1}.$$

Substituting this expression into the Taylor rule yields

$$i_t = \frac{\phi_\pi}{\beta}\mathbb{E}_t \pi_{t+1} - \frac{\phi_\pi \kappa}{\beta}\mathbb{E}_t y_{t+1} + e_t^i.$$

Inserting this policy rule into the IS equation and rearranging terms gives

$$y_t = \left(1 - \frac{\phi_\pi}{\beta}\right)\mathbb{E}_t \pi_{t+1} + \left(1 + \frac{\phi_\pi \kappa}{\beta}\right)\mathbb{E}_t y_{t+1} - e_t^i. \tag{53}$$

Together with the New Keynesian Phillips curve, this equation characterizes the dynamics of inflation and output under a forward-looking Taylor rule with $k = 2$.

Determinacy. Again, the IS equation and the Phillips curve form a linear system in (π_t, y_t). Writing the system in matrix form as

$$A_2 \mathbb{E}_t z_{t+1} = B_2 z_t + C_2 e_t, \qquad z_t = (\pi_t, y_t)', \tag{54}$$

the matrices A_2 and B_2 are given by

$$A_2 = \begin{pmatrix} 1 - \dfrac{\phi_\pi}{\beta} & 1 + \dfrac{\phi_\pi \kappa}{\beta} \\ \beta & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 0 & 1 \\ 1 & -\kappa \end{pmatrix}. \tag{55}$$

The matrix A_2 is invertible for all admissible parameter values. The reduced-form transition matrix is therefore given by $A_2^{-1}B_2$. The transmission matrix is:

$$A_2^{-1}B_2 = \begin{pmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ -\frac{c_2}{\beta d} & \frac{1}{d} + \frac{\kappa c_2}{\beta d} \end{pmatrix} \tag{56}$$

with

$$c_2 = 1 - \frac{\phi_\pi}{\beta}, \quad d = 1 + \frac{\phi_\pi \kappa}{\beta}.$$

Evaluating the same determinacy conditions as in the $k = 1$ case yields

$$1 < \phi_\pi < \frac{1 - \beta}{\kappa}. \tag{57}$$

Under standard parameter calibrations, $(1 - \beta)/\kappa < 1$, so that the two inequalities cannot be satisfied simultaneously. As a result, the monetary policy block can generate at most one unstable eigenvalue for two forward-looking variables. The Blanchard–Kahn conditions therefore fail, and equilibrium determinacy fails for $k = 2$, unless κ is very small.

C Indeterminacy in the general case ($\mathbf{k > 2}$)

The matrices A_k, B_k and C_k are given by:

$$A_k = \begin{pmatrix} 1 - \dfrac{\phi_\pi}{\beta^{k-1}} & 0 & \dfrac{\phi_\pi \kappa}{\beta^{k-2}} & \dfrac{\phi_\pi \kappa}{\beta^{k-3}} & \cdots & \dfrac{\phi_\pi \kappa}{\beta} \\ \beta & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, B_k = \begin{pmatrix} 0 & 1 & -1 - \dfrac{\phi_\pi \kappa}{\beta^{k-1}} & 0 & \cdots & 0 \\ 1 & -\kappa & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, C_k = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

The transition matrix $A_k^{-1}B_k$ has a companion-type structure:

$$A_k^{-1}B_k = \begin{pmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ -\frac{c_k}{\beta d} & \frac{\beta + c_k\kappa}{\beta d} & -\frac{\beta^{k-2}+d}{\beta^{k-2}d} & -\frac{1}{\beta^{k-3}} & -\frac{1}{\beta^{k-4}} & \cdots & -\frac{1}{\beta} \end{pmatrix}, \tag{58}$$

with

$$c_k \equiv 1 - \frac{\phi_\pi}{\beta^{k-1}}, \qquad d \equiv 1 + \frac{\phi_\pi \kappa}{\beta},$$

yielding the characteristic polynomial (13).

For the special case of $k = 3$, the matrices A_3, B_3, and C_3 are given by

$$A_3 = \begin{pmatrix} 1 - \frac{\phi_\pi}{\beta^2} & 0 & \frac{\phi_\pi \kappa}{\beta} \\ \beta & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 0 & 1 & -\frac{\phi_\pi \kappa}{\beta^2} - 1 \\ 1 & -\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad C_3 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}. \tag{59}$$

The matrix A_3 is invertible for all admissible parameter values. The transmission matrix is given by

$$A_3^{-1}B_3 = \begin{pmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ 0 & 0 & 1 \\ -\frac{c_3}{\beta d} & \frac{\beta + c_3\kappa}{\beta d} & -\frac{\beta + d}{\beta d} \end{pmatrix}, \quad \text{with} \quad c_3 = 1 - \frac{\phi_\pi}{\beta^2}, \quad d = 1 + \frac{\phi_\pi \kappa}{\beta}, \tag{60}$$

yielding the characteristic polynomial (14).

D Jury (1962)'s stability test

Jury's stability test provides necessary and sufficient conditions for all roots of a polynomial to lie strictly inside the unit circle. Since determinacy in our model requires all roots of the characteristic polynomial to lie outside the unit circle, we apply the test to the reciprocal polynomial:

$$Q_3(\mu) = \mu^3 - (\beta + \kappa + 1)\mu^2 + \beta\mu + \phi_\pi \kappa. \tag{61}$$

Requiring all roots of $Q_3(\mu)$ to lie inside the unit circle is equivalent to requiring all roots of the original characteristic polynomial to lie outside the unit circle because $|\lambda_j| > 1 \iff |\mu_j| < 1$. For a cubic polynomial of the form $Q(\mu) = \mu^3 + a_1\mu^2 + a_2\mu + a_3$, the Jury conditions are

$$1 + a_1 + a_2 + a_3 > 0, \tag{62}$$

$$1 - a_1 + a_2 - a_3 > 0, \tag{63}$$

$$1 - a_3 > 0, \tag{64}$$

$$1 - a_2 + a_1 a_3 - a_3^2 > 0. \tag{65}$$

For the polynomial (61), condition (62) becomes $\kappa(\phi_\pi - 1) > 0$. Since $\kappa > 0$, the inequality requires

$$\phi_\pi > 1.$$

Thus, the Jury condition is satisfied to the right of the unit-circle crossing at $\phi_\pi = 1$, establishing this crossing as a lower bound on the determinacy region. If we define

$$g(\phi_\pi) = 1 - \beta - (\beta + \kappa + 1)\phi_\pi\kappa - (\phi_\pi\kappa)^2,$$

condition (65) yields $g(\phi_\pi) > 0$. Taking the first derivative yields

$$g'(\phi_\pi) = -\kappa(\beta + \kappa + 1) - 2\phi_\pi\kappa^2,$$

which is negative since $\beta > 0, \kappa > 0, \phi_\pi > 0$. That is, $g(\phi_\pi)$ is strictly decreasing in $\phi_\pi > 0$ and equals zero at $\phi_\pi = \bar{\phi}_\pi(3, \beta, \kappa)$. Hence, for $\phi_\pi < \bar{\phi}_\pi(3, \beta, \kappa)$, condition (65) is satisfied to the left of the complex-conjugate unit-circle crossing, establishing $\phi_\pi = \bar{\phi}_\pi(3, \beta, \kappa)$ as an upper bound on the determinacy region. Conditions (63) and (64) impose no additional restrictions on the determinacy region, as the corresponding upper bounds lie above the upper bound implied by condition (65). Taken together, these conditions imply that a non-empty determinacy region without the fiscal block can exist only if

$$1 < \phi_\pi < \bar{\phi}_\pi(3, \beta, \kappa).$$

E Welfare Derivation

The minimization problem is:

$$\min \mathcal{L}(k, \phi_\pi) = \min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t(\pi_t^2 + \zeta y_t^2),$$

subject to

$$y_t = \mathbb{E}_t y_{t+1} - i_t + \mathbb{E}_t \pi_{t+1}, \qquad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t,$$
$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+k}, \qquad u_t = \rho_u u_{t-1} + \epsilon_t^u.$$

The MSV solution is guessed to take the form: $\pi_t = a_\pi(k, \phi_\pi) u_t$, and $y_t = a_y(k, \phi_\pi) u_t$. Since u_t is AR(1), we have:

$$\mathbb{E}_t \pi_{t+j} = a_\pi \rho_u^j u_t, \qquad \mathbb{E}_t y_{t+j} = a_y \rho_u^j u_t.$$

Substituting this into the three equations yields

$$\begin{aligned} \text{Taylor rule}: &\quad i_t = \phi_\pi a_\pi \rho_u^k u_t, \\ \text{IS curve}: &\quad a_y u_t = a_y \rho_u u_t - \phi_\pi a_\pi \rho_u^k u_t + a_\pi \rho_u u_t, \\ \text{NKPC}: &\quad a_\pi u_t = \beta a_\pi \rho_u u_t + \kappa a_y u_t + u_t. \end{aligned}$$

This gives, as guessed, a two-equation system for a_π and a_y.

$$a_y = \frac{(\rho_u - \phi_\pi \rho_u^k) a_\pi}{1 - \rho_u}, \qquad a_\pi = \frac{\kappa a_y + 1}{1 - \beta \rho_u}.$$

Assuming that the cost-push shock is initially drawn from its stationary distribution, its unconditional variance is

$$\mathbb{E}[u_t^2] = \frac{\sigma_\epsilon^2}{1 - \rho_u^2}.$$

Hence:

$$\mathcal{L} = \frac{1}{2}(a_\pi^2 + \zeta a_y^2) \frac{\sigma_\epsilon^2}{(1 - \rho_u^2)(1 - \beta)}$$

so the ranking of alternative policy rules depends on $\bar{\mathcal{L}}(k, \phi_\pi) = a_\pi^2 + \zeta a_y^2$, which itself depends only on the policy parameters and admits an analytical characterization. The next analytical step is to minimize the MSV loss with respect to ϕ_π, for a given forecast horizon k. We can define:

$$m(k, \phi_\pi) \equiv \frac{\rho_u - \phi_\pi \rho_u^k}{1 - \rho_u}.$$

Then $a_y = m a_\pi$, where m measures how much the output gap moves relative to inflation after a cost-push shock, and the loss-relevant object is:

$$\tilde{\mathcal{L}}(k, \phi_\pi) = a_\pi^2 (1 + \zeta m^2).$$

Next, we minimize

$$\min_{\phi_\pi} \tilde{\mathcal{L}}(k, \phi_\pi)$$

for each given k. This gives the MSV-optimal inflation response before imposing determinacy. Plugging

$$a_\pi(m) = \frac{1}{1 - \beta\rho_u - \kappa m}, \qquad a_y(m) = \frac{m}{1 - \beta\rho_u - \kappa m}$$

into the loss function yields:

$$\tilde{\mathcal{L}}(m) = \frac{1 + \zeta m^2}{(1 - \beta\rho_u - \kappa m)^2}.$$

Hence, the optimization problem reduces to choosing the optimal inflation-output trade-off parameter m. Now set:

$$\frac{\partial \mathcal{L}}{\partial m} = 0,$$

which results in (observing $\zeta = \kappa/\epsilon$)

$$m^* = -\frac{\epsilon}{1 - \beta\rho_u}$$

This is the optimal inflation-output tradeoff. Mapping this back to ϕ_π:

$$m^* = \frac{\rho_u - \phi_\pi \rho_u^k}{1 - \rho_u} \iff \phi_\pi = \frac{\rho_u - m^*(1 - \rho_u)}{\rho_u^k}$$

and inserting the optimal m^* gives

$$\phi_\pi^*(k) = \frac{\rho_u + \epsilon(1 - \rho_u)/(1 - \beta\rho_u)}{\rho_u^k}. \tag{66}$$

F Determinacy with Fiscal Policy

The matrices A_{k+1}, B_{k+1} and C_{k+1} take the form

$$A_{k+1} = \begin{pmatrix} 1 & -\frac{\gamma\phi_\pi}{\beta^{k-1}} & 0 & -\frac{\gamma\phi_\pi\kappa}{\beta^{k-1}} & \cdots & -\frac{\gamma\phi_\pi\kappa}{\beta^2} \\ 0 & 1 - \frac{\phi_\pi}{\beta^{k-1}} & 0 & \frac{\phi_\pi\kappa}{\beta^{k-2}} & \cdots & \frac{\phi_\pi\kappa}{\beta} \\ 0 & \beta & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix},$$

$$B_{k+1} = \begin{pmatrix} \frac{1-\alpha_b}{\beta} & -\frac{\gamma}{\beta} & -\frac{\alpha_y}{\beta} & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 - \frac{\phi_\pi \kappa}{\beta^{k-1}} & \cdots & 0 \\ 0 & 1 & -\kappa & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \qquad C_{k+1} = \begin{pmatrix} \gamma & \frac{1}{\beta}(\alpha_b - 1) \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}.$$

G Inflation-Inertia Threshold for Determinacy

We characterize the minimum degree of inflation inertia required to restore determinacy for an arbitrary inflation forecast horizon k. Throughout, we consider the hybrid New Keynesian Phillips curve

$$\pi_t = (1 - \rho_\pi)\beta E_t \pi_{t+1} + \kappa y_t + \rho_\pi \pi_{t-1}, \tag{67}$$

combined with the forward-looking Taylor rule $i_t = \phi_\pi E_t \pi_{t+k} + e_t^i$.

Characteristic polynomial. For $k \geq 3$, the associated transition matrix implies the characteristic polynomial

$$P_k(\lambda, \rho_\pi) = \lambda^{k+1} + \frac{\beta(1-\rho_\pi)}{\phi_\pi \kappa}\lambda^3 - \frac{\beta(1-\rho_\pi) + \kappa + 1}{\phi_\pi \kappa}\lambda^2 + \frac{1+\rho_\pi}{\phi_\pi \kappa}\lambda - \frac{\rho_\pi}{\phi_\pi \kappa}. \tag{68}$$

As in the baseline model, extending the target horizon affects the characteristic polynomial only through the leading term λ^{k+1}. Inflation inertia therefore does not alter the source of instability associated with longer inflation forecast horizons. Instead, it modifies the lower-order coefficients of the polynomial via the parameter ρ_π, thereby shifting the eigenvalues. Consequently, sufficiently strong inflation persistence may restore determinacy for moderate forecast horizons. Multiplying (68) by $\phi_\pi \kappa$ and collecting terms in ρ_π yields

$$\rho_\pi(\lambda, k) = \frac{\phi_\pi \kappa \lambda^{k+1} + \beta\lambda^3 - (\beta + \kappa + 1)\lambda^2 + \lambda}{\beta\lambda^3 - \beta\lambda^2 - \lambda + 1}. \tag{69}$$

To characterize the determinacy threshold, we again employ a unit-circle argument. Evaluating (69) at

$$\lambda = e^{i\theta}, \qquad \theta \in (0, \pi),$$

yields

$$\rho_\pi(\lambda, k) = \frac{\mathcal{N}(\theta)}{\mathcal{D}(\theta)}, \tag{70}$$

with

$$\mathcal{N}(\theta) := \phi_\pi \kappa e^{i(k+1)\theta} + \beta e^{i3\theta} - (\beta + \kappa + 1)e^{i2\theta} + e^{i\theta}, \tag{71}$$

$$\mathcal{D}(\theta) := \beta e^{i3\theta} - \beta e^{i2\theta} - e^{i\theta} + 1. \tag{72}$$

Since ρ_π is real-valued, admissible solutions require the right-hand side of (70) to be real. Using Euler's formula, $e^{im\theta} = \cos(m\theta) + i\sin(m\theta)$, allows us to express (71) and (72) in terms of their real and imaginary components. Write $\mathcal{N}(\theta) = \mathcal{N}_r(\theta) + i\mathcal{N}_i(\theta)$, and $\mathcal{D}(\theta) = \mathcal{D}_r(\theta) + i\mathcal{D}_i(\theta)$, where

$$\begin{aligned}
\mathcal{N}_r(\theta) &= \phi_\pi \kappa \cos([k+1]\theta) + \beta\cos(3\theta) - (\beta + \kappa + 1)\cos(2\theta) + \cos\theta, \\
\mathcal{N}_i(\theta) &= \phi_\pi \kappa \sin([k+1]\theta) + \beta\sin(3\theta) - (\beta + \kappa + 1)\sin(2\theta) + \sin\theta, \\
\mathcal{D}_r(\theta) &= \beta\cos(3\theta) - \beta\cos(2\theta) - \cos\theta + 1, \\
\mathcal{D}_i(\theta) &= \beta\sin(3\theta) - \beta\sin(2\theta) - \sin\theta.
\end{aligned}$$

Multiplying the numerator and denominator by the complex conjugate of the denominator, $(\mathcal{D}_r - i\mathcal{D}_i)$, gives

$$\frac{\mathcal{N}_r + i\mathcal{N}_i}{\mathcal{D}_r + i\mathcal{D}_i} = \frac{(\mathcal{N}_r\mathcal{D}_r + \mathcal{N}_i\mathcal{D}_i) + i(\mathcal{N}_i\mathcal{D}_r - \mathcal{N}_r\mathcal{D}_i)}{\mathcal{D}_r^2 + \mathcal{D}_i^2}.$$

Therefore, the condition that (70) be real is equivalent to

$$\mathcal{N}_i(\theta)\,\mathcal{D}_r(\theta) - \mathcal{N}_r(\theta)\,\mathcal{D}_i(\theta) = 0. \tag{73}$$

Substituting the expressions above yields

$$\begin{aligned}
&\Big[\phi_\pi \kappa \sin([k+1]\theta) + \beta\sin(3\theta) - (\beta + \kappa + 1)\sin(2\theta) + \sin\theta\Big]\Big[\beta\cos(3\theta) - \beta\cos(2\theta) - \cos\theta + 1\Big] \\
&= \Big[\phi_\pi \kappa \cos([k+1]\theta) + \beta\cos(3\theta) - (\beta + \kappa + 1)\cos(2\theta) + \cos\theta\Big]\Big[\beta\sin(3\theta) - \beta\sin(2\theta) - \sin\theta\Big].
\end{aligned} \tag{74}$$

Equation (74) determines the candidate frequencies $\theta \in (0, \pi)$ for which the value of ρ_π implied by a unit-circle root $\lambda = e^{i\theta}$ is real. For each solution $\theta^* \in (0, \pi)$, the corresponding boundary value of inflation inertia is obtained by substituting θ^* back into the real part of (69). The determinacy threshold can therefore be characterized as

$$\rho_\pi^{\min}(k, \beta, \kappa, \phi_\pi) = \min_{\theta^* \in \Theta} \rho_\pi(\theta^*; k), \quad \text{where} \quad \Theta = \{\theta \in (0, \pi) : (74) \text{ holds} \quad \text{and} \quad \rho_\pi \in [0, 1]\}.$$

In general, this threshold does not admit a closed-form representation and is therefore evaluated numerically.