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DEPARTMENT OF ECONOMICS

Legislative bargaining with private information: A comparison of majority and unanimity rule

David Piazolo

Christoph Vanberg

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Abstract

We present a three-person, two-period bargaining game with private information. A single proposer is seeking to secure agreement to a proposal under either majority or unanimity rule. If the first period proposal fails, the game ends immediately with an exogenously given "breakdown" probability. Two responders have privately known disagreement payoffs. We characterize Bayesian equilibria in stagewise undominated strategies. Our central result is that responders have a signaling incentive to vote "no" on the first proposal under unanimity rule, whereas no such incentive exists under majority rule. The reason is that being perceived as a "high breakdown value type" is advantageous under unanimity rule, but disadvantageous under majority rule. As a consequence, responders are "more expensive" under unanimity rule and disagreement is more likely. These results confirm intuitions that have been stated informally before and in addition yield deeper insights into the underlying incentives and what they imply for optimal behavior in bargaining with private information.

JEL codes: C78; D72; D82

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*Heidelberg University, david.piazolo@awi.uni-heidelberg.de

†Corresponding author, Heidelberg University, vanberg@uni-heidelberg.de

1 Introduction

A fundamental problem in Political Economics is the choice between alternative q-majority rules. In seminal work, Buchanan and Tullock (1965) analyzed this choice from a constitutional perspective. They argued, largely informally, that unanimity rule has the advantage of ensuring that all decisions constitute Pareto improvements, but the disadvantage that it may be associated with large ”decision costs”. One reason is that unanimity rule may induce incentives for individuals to “act tough” during negotiations. For example, bargainers may attempt to commit to a tough stance, or *pretend* to be opposed to agreement, hoping to secure concessions from others. Such incentives may be mitigated by the use of less-than-unanimity rules.

Certain aspects of these ideas can be formalized using models involving perfect information (see, e.g., Miettinen and Vanberg (2024)). However, the core intuition as well as its real-world counterpart clearly involve *private information* about individual inclinations to support a proposal. In the presence of private information, the actions taken by individual bargainers may act as (costly) *signals* of their privately known preferences. Our aims in this paper are to model the role of private information explicitly, in order to investigate how such *signaling incentives* are affected by the decision rule being used, and to gain a deeper understanding of what they imply about optimal behavior by proposers and responders in bargaining with private information. In particular, our focus will be on the *signaling value* of voting to support or reject a proposal and how it is affected by the decision rule and the type of proposal being considered.

We analyze a two-period, three-player bargaining game involving a fixed proposer who is trying to secure agreement to a reform, either under majority ($q = 2$) or unanimity ($q = 3$) rule. The two responders can be of two different types, who differ in their willingness to agree to the reform. This difference is modeled by endowing each responder with a privately known “breakdown value” (either “high” or “low”), which they receive if no agreement is reached. A natural interpretation is that a player’s “breakdown value” reflects the utility of the “status quo” which will prevail if the reform is not implemented. The proposer is endowed with a single monetary unit that she can use to “buy” the responders’ votes, and she has (at most) two chances to get a proposal passed. The monetary unit can be interpreted as a transferable surplus that will be generated by the proposed change to the “status quo”. Breakdown occurs with some probability if the first proposal fails, and for sure if the second proposal fails. (The game is formally defined in Section 3.) Despite its simplicity, this game is very rich in strategic complexity. This is due to the endogenous signaling effects associated with voting behavior. Our equilibrium concept is Perfect Bayesian Equilibrium in stagewise undominated strategies.

A key insight emerging from the analysis is that responders consider two types of incentives when choosing how to vote on a given proposal. The first relates to the influence of their vote on the fate of the proposal under consideration. The second pertains to its effect on the proposer's belief about their type. Under some circumstances, these incentives may point in opposite directions. That is, a responder may prefer that a given proposal passes, but fear that voting "yes" will reveal something about their type which they would rather conceil. The relative importance of these considerations depends on the probability with which the responder's vote is pivotal. The strategic implications are quite intricate, and play out quite differently under majority and unanimity rule.

Our main results are as follows. Under both majority and unanimity rule, voting "no" on certain first period proposals constitutes a *signal* that a responder is a "high" type. Under unanimity rule, this signaling effect benefits the responder in the sense that it causes the proposer to make a more favorable offer to him in Period 2. We refer to this as a positive "signaling value" from rejection. Under majority rule, by contrast, the signaling value is generally *negative*, because responders who vote "no" can be excluded from subsequent coalitions (and it is in the interest of the proposer to do so). As a consequence of this difference in signaling incentives, responders are "more expensive" (in a sense that will be made more precise) under unanimity rule. The set of proposals that can pass in Period 1 is significantly larger under majority rule, not just because agreement requires fewer "yes"-votes, but also because responders are more willing to vote "yes" when allocated a given share. Under majority rule, agreement in the first period is more likely in the sense that, for a wide parameter range, the proposer will find it optimal to immediately secure one responder's agreement, while under unanimity rule, she prefers to make a proposal that fails with positive probability.

Overall, these results lend support to the intuitive argument that unanimity rule induces "tough" bargaining behavior and is associated with greater delay. In addition, our analysis provides a deeper understanding of the signaling incentives in multilateral bargaining as well as implications for optimal behavior in the presence of private information. The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents our game and equilibrium concept. Sections 4 and 5 present analyses of majority and unanimity rule, respectively. Section 6 compares the results in order to make precise our conclusions that unanimity rule is associated with higher "prices" and more delay. Section 7 concludes and discusses the additional strategic insights generated by the analysis. Proofs are presented in the Appendix.

2 Related Literature

Our contribution fits broadly into a literature on q-majority rules in multilateral bargaining games with complete information. More specifically, there is a connection to models that involve any sort of heterogeneity with respect to individual players' willingness to agree. For example, consider a standard Baron and Ferejohn (1989) bargaining game with complete information and heterogeneous discount factors. There, a larger discount factor implies a "tougher" bargaining stance and a higher "price" for a player's vote. It is folk knowledge that this is advantageous under unanimity rule but not under majority rule. That is, a player's expected payoff is increasing in his discount factor under unanimity rule, but non-monotone (and eventually decreasing) under less-than-unanimity rules. Miller et al. (2018) consider a modified Baron-Ferejohn game with *commonly known* breakdown values and show that a larger breakdown value is advantageous under unanimity rule, but can be bad under majority rule because "expensive" players are excluded from winning coalitions. These results are *suggestive* of the idea that, when the source of heterogeneity is private information, players would like to *signal* that they are "tough" (e.g., have a large discount factor or breakdown value) under unanimity rule, while they would not like to do so under less-than-unanimity rules.

The literature on multilateral bargaining with private information is scarce. Tsai and Yang (2010) investigate a three-player, three-period Baron-Ferejohn game with private information about discount factors. They show that under majority rule, equilibrium might involve delay and oversized coalitions. An important difference between our paper and Tsai and Yang (2010) is that we also investigate unanimity rule, allowing us to discuss the different signaling incentives associated with different voting rules.[1]

Chen and Eraslan (2014) investigate a three-player majoritarian bargaining game over an ideological and distributive decision with private information about ideological intensities. As in our setting, a single proposer is seeking the support of either one (majority rule) or both (unanimity rule) responders, and she is unsure as to how much each responder must be paid in order to secure his vote. Unlike in our setting, the proposer can make only one proposal, so there is no room for costly signaling by voting "no". Instead, Chen and Eraslan (2014) consider cheap talk communication prior to the proposal stage. They investigate the circumstances under which cheap talk communication can credibly convey information. A central finding is that competition between the two responders may result in less information conveyed in equilibrium and may make the proposer worse off as compared to unanimity rule (and only one responder). (Chen and Eraslan (2013)

[1] Tsai (2009) discusses a similar game under both majority and unanimity rule, deriving conditions for the existence of separating and pooling equilibria. Our model differs in several respects, and we provide a significantly more comprehensive equilibrium analysis.

investigate a similar model in which the responders' ideal points are private information.)

Our paper is also related to the literature on reputational bargaining. Abreu and Gul (2000) investigate a bilateral bargaining game in which players may, with some probability, be of an "obstinate" type that is committed to claim a certain share of the pie. They show that the presence of such types creates incentives for "rational" types to imitate them, i.e., to "act tough". Ma (2023) applies this idea to a multilateral *majoritarian* bargaining game and finds that the threat of exclusion under majority rule removes this incentive to act "tough".

Finally, our paper is also related to the literature in *bilateral* bargaining with private information. Naturally, these models only consider unanimity rule ($q = n = 2$). The paper that is most relevant to our analysis is Fudenberg and Tirole (1983), who investigate a two-person, two-period bargaining game in which a seller is looking to sell an object to a buyer whose valuation is privately known. The seller has two chances to make a proposal (suggested price). As in their model, equilibrium strategies in our context necessarily involve buyers *mixing* between accepting and rejecting certain proposals.

3 Bargaining Game and Equilibrium Concept

The bargaining game The game involves three players, a Proposer (P, she) and two responders (R1 and R2, he). The game lasts two periods $t = 1, 2$. Each period consists of a *proposal stage* and a *voting stage*. During the proposal stage, P makes a *proposal* $x_t = (x_{1t}, x_{2t}) \in X$, where $X = \{(x_1, x_2) : x_i \geq 0, x_1 + x_2 \leq 1\}$. Then R1 and R2 simultaneously *vote* either "yes" (Y) or "no" (N), denoted $v_{it} \in \{Y, N\}$. If q responders vote Y, the proposal *passes*. In this case the payoffs are $\pi_P = 1 - x_{1t} - x_{2t}$ and $\pi_i = x_{it}$ for $i = 1, 2$. If fewer than q responders vote Y, the proposal *fails*. If the Period 1 proposal fails, the game ends in "breakdown" with probability $(1 - \delta)$ and otherwise continues to Period 2. If the Period 2 proposal fails, the game ends in breakdown. In case of breakdown in either period, P's payoff is $\pi_P = 0$ and Ri's payoff is $\pi_i = b_i$, where b_i denotes an exogenously given "breakdown value".

Responders can be of two types, which we denote $T \in \{L, H\}$ ("low", "high"). The "low" type's breakdown value is denoted $l > 0$, and the "high" type's is $h > l$. Thus, the high type is more strongly opposed to agreement, and presumably requires a larger compensation in return for a "yes" vote. For some of what follows, it will be convenient to define $\tau = h - l$ to be the difference in high and low breakdown values. Types are independently drawn and equally likely ex ante.

Throughout the analysis, we assume that agreement is *efficient* even in the "worst case" where both responders are of type H. This requires $h < \frac{1}{2}$ (implying $\tau < \frac{1-2l}{2}$). This assumption implies that P is willing to offer both responders h in Period 2 if she is sufficiently sure that they are both of type H.[2] Note that payoffs are not discounted, but delay is costly due to the probability of breakdown.

Strategies A pure strategy for P specifies a Period 1 proposal $x_1 = (x_{11}, x_{21})$ as well as a Period 2 proposal following any history that leads to Period 2. Such a history consists of a Period 1 proposal and voting patterns $v_1 = (v_{11}, v_{21}) \in \{Y, N\}^2$, where v_{i1} denotes Responder i's vote on the first proposal. If $s \in X \times \{Y, N\}^2$ is a history leading to failure of the first proposal, we denote P's ensuing Period 2 proposal by $x_2(s)$. For example, $x_2\left((\frac{1}{2}, 0), (N, N)\right)$ is the proposal made following a history where $(\frac{1}{2}, 0)$ was proposed in Period 1 and both responders voted N.

A strategy for Responder i specifies type-dependent acceptance probabilities $\mu_{it}^T(x) \in [0,1]$ for all $x \in X$ and $t = 1, 2$.[3] For some parts of the analysis, we will drop the dependence on both t and x (when both are considered fixed), and write Ri's acceptance probabilities as $\mu_i = \left(\mu_i^L, \mu_i^H\right)$. For example, $\mu_1 = (1, 0)$ means that R1 votes Y if his breakdown value is low and N if his breakdown value is high.

Equilibrium concept Our equilibrium concept is perfect Bayesian equilibrium in stagewise undominated strategies, with an intuitive restriction on off-path beliefs. In the following definition, a "history" is a complete description of all proposals and votes that occur before an information set at which a given player is called upon to take an action.

Definition 1. A strategy profile $(\sigma_1, \sigma_2, \sigma_P)$ and (common) belief system ω is an *equilibrium* if the action specified by each σ_i at each history maximizes i's payoff given beliefs at that history and in addition, the following criteria are met:

1. In all stages of the game involving simultaneous actions, strategies place zero probability on actions which are weakly dominated given continuation payoffs implied by equilibrium play in later stages.

2. Beliefs are formed using Bayes' rule whenever possible.

3. Off-path beliefs satisfy:

[2] Admittedly, this assumption is restrictive, as it precludes an analysis of the substantively interesting case where agreement may be inefficient. However, as will become clear, the analysis is quite involved and involves several case distinctions even under this simplifying assumption. Thus the more general case is left for future research.

[3] In principle, Period 2 acceptance probabilities could depend on elements of the history other than the proposal x_2. However, elimination of weakly dominated strategies implies that in equilibrium they will not. It is therefore without loss of generality to write acceptance probabilities as functions of x and T only.

(a) If receiver i votes N following a Period 1 proposal at which his equilibrium strategy is $\mu_i = (1, 1)$ (i.e., following an unexpected N-vote), beliefs assign probability 1 to that responder being a high type.

(b) If receiver i votes Y following a Period 1 proposal at which his equilibrium strategy is $\mu_i = (0, 0)$ (i.e., following an unexpected Y-vote), beliefs assign probability 0 to that responder being a high type.

Outline of analysis We begin by providing a comprehensive analysis of all "continuation equilibria" following any first round proposal. It will turn out that many proposals admit multiple continuation equilibria. The reason is that voting in Period 1 determines Period 2 beliefs, which in turn determine the Period 2 proposal, inducing expectations that rationalize the Period 1 voting patterns in a self fulfilling loop.

To deal with this multiplicity, we focus on those continuation equilibria which the proposer most prefers. This approach can be motivated by imagining the proposer making a self-enforcing announcement as to his intentions in the event of proposal failure. Finally, we identify the optimal first round proposal given this refinement.

4 Majority rule

We analyze the game by backward induction, beginning with the final vote (Lemma 1) and the second period proposal.

4.1 Period 2 voting and proposals

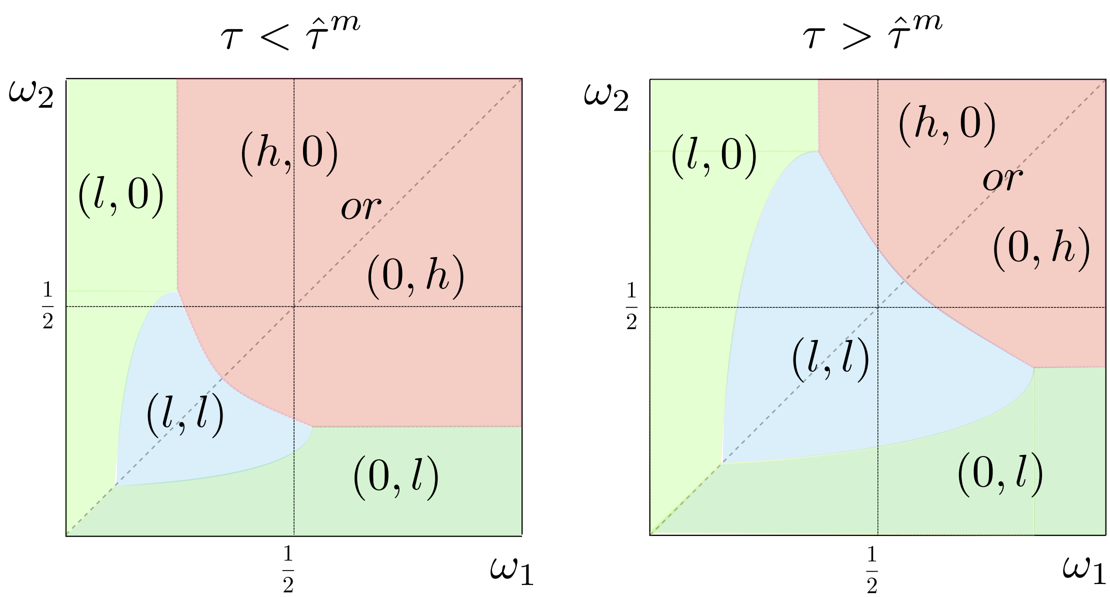
Since the game ends in breakdown if the Period 2 proposal fails, responders compare what they are offered to their breakdown value. Voting insincerely is weakly dominated, and it is without loss of generality to assume that responders vote Y when indifferent. This establishes the following.

Lemma 1. *In any equilibrium, responders vote Y in Period 2 iff $x_{i2} \geq b_i$, that is iff they are offered at least their breakdown value.*

In Period 2, three types of proposals can conceivably be optimal: Offering h to one responder (and zero to the other) ensures agreement irrespective of their type. Offering l to one responder passes only if the respective responder is a low type. Offering l to both passes if at least one is a low type. Which of these options is optimal will depend on P's beliefs.

Let $\omega = (\omega_1, \omega_2)$ denote P's belief concerning the probabilities that R1 and R2 are high types. (This will depend on the prior history, but is treated as given at this stage of the analysis.) Figure 1 graphically depicts the optimal second period proposal as a function of these beliefs. (See Appendix A.1 for the derivation and functional form.)

Figure 1: Period 2 Proposals (majority rule)



The figure depicts the Period 2 proposals that P makes given arbitrary beliefs about the responders' types. The horizontal and vertical axes measure the probability that R1 and R2 are of the high type, denoted ω_1 and ω_2 respectively. The left and right panels distinguish the cases where $\tau = h - l$ is greater or smaller than $\hat{\tau}^m \equiv \frac{1+2l}{4}$, which determines the optimal proposal in case $\omega_1 = \omega_2 = \frac{1}{2}$.

Note that the Period 2 proposal is qualitatively affected by whether the difference between the two types' disagreement payoffs, denoted $\tau = h - l$, is small or large. In the following analysis, it will be useful to define a threshold for τ at which this qualitative change occurs. We thus define

$$\hat{\tau}^m \equiv \frac{1+2l}{4}.$$

Suppose for a moment that no information is revealed prior to Period 2, so that $\omega = (\frac{1}{2}, \frac{1}{2})$. Then if $\tau < \hat{\tau}^m$, P prefers to offer one responder h in order to make sure that her proposal passes. If $\tau > \hat{\tau}^m$ (i.e., low types are significantly cheaper), P prefers to "gamble" by offering *both* responders l, which passes with probability $\frac{3}{4}$.

4.2 Continuation values and beliefs

Let $u_i^T(s)$ denote Ri's expected utility in Period 2, following a history s, and given his type T. In general, this will depend on the beliefs $\omega = (\omega_1, \omega_2)$ induced by the prior history. For example, if $\tau > \hat{\tau}^m$, then any history that induces beliefs $\omega = (\frac{1}{2}, \frac{1}{2})$ will be followed by the proposal $x_2 = (l, l)$. Therefore, the expected utility of a low type responder at such histories is l (which he receives irrespective of whether the proposal passes or not), while the expected utility of a high type responder is $\frac{l+h}{2}$, as she will vote N and the proposal will pass if and only if the *other* responder is low. Similar reasoning applies for histories inducing other beliefs.[4]

Under majority rule, Period 2 is reached only if *both* responders voted N in Period 1. Then, provided that the Period 1 acceptance probabilities are $(\mu_i^L, \mu_i^H) \neq (1, 1)$, the Period 2 belief about Responder i (given that he voted N) is given by

$$\omega_i^N = \frac{1 - \mu_i^H}{2 - \mu_i^H - \mu_i^L}$$

In case $(\mu_i^L, \mu_i^H) = (1, 1)$, beliefs following an "unexpected" (off path) N-vote are not determined by Bayes' rule. As noted above, we assume in this case that $\omega_i = 1$. The following Lemma is stated without proof.

Lemma 2. *In any equilibrium and following any history s, Responder i's Period 2 expected payoff under majority rule satisfies*

- $u_i^T(s) \in [0, h]$ *for* $T \in \{L, H\}$, *and*
- $u_i^H(s) \geq u_i^L(s)$.

4.3 Period 1 voting

Fix an arbitrary Period 1 proposal (x_1, x_2), and consider how Responder i may vote on this proposal in what we will call a "continuation equilibrium". Under majority rule, his vote makes a difference only in the event that the other responder votes N. In this case, voting Y yields x_i, while voting N yields $(1-\delta) b_i + \delta u_i^T(s)$, where s is the history consisting of proposal (x_1, x_2) and voting pattern (N, N). Therefore, voting Y is weakly dominated if $x_i < (1-\delta) b_i + \delta u_i^T(s)$, while voting N is weakly dominated if $x_i > (1-\delta) b_i + \delta u_i^T(s)$. Then the following lemma follows directly from the fact that $u_i^T(s) \in [0, h]$.

[4] For some values of ω, P is indifferent between multiple proposals. As a consequence, such ω are consistent with an entire *set* of continuation values consistent with P mixing between those proposals. In addition, *different* histories leading to such a belief may be associated with different continuation values, as P's proposal (mix) could depend on the history itself.

Lemma 3. *Let* (x_1, x_2) *be an arbitrary Period 1 proposal. Then in any equilibrium under majority rule,*

- *if* $x_i \geq h$, *then* $\mu_i^L = \mu_i^H = 1$,
- *if* $x_i \geq l + \delta\tau$, *then* $\mu_i^L = 1$,
- *if* $x_i < (1-\delta)\,h$, *then* $\mu_i^H = 0$, *and*
- *if* $x_i < (1-\delta)\,l$, *then* $\mu_i^L = \mu_i^H = 0$.

Proof. All strict inequalities follow from elimination of weakly dominated strategies. The weak inequalities in cases (1) and (2) are without loss of generality, since an arbitrarily small increment in the offer would guarantee acceptance. □

A Corollary to Lemma 3 is that a low type is always at least as likely to vote Y as a high type, with implications for the signaling consequences of voting N.

Corollary 1. *In any equilibrium under majority rule, for any Period 1 proposal* (x_1, x_2), $\mu_i^L \geq \mu_i^H$. *In particular,*

- *if* $\mu_i^H > 0$, *then* $\mu_i^L = 1$, *and*
- *if* $\mu_i^L < 1$, *then* $\mu_i^H = 0$.

Thus $\omega_i^N \geq \frac{1}{2}$.

Proof. Suppose $\mu_i^H > 0$. Then voting Y is not weakly dominated for a high type, which implies that voting N is weakly dominated for the low type. Analogous reasoning establishes the second observation. The last obviously follows from the other two. □

Another implication of Lemma 3 is the following.

Corollary 2. *In any equilibrium under majority rule, any Period 1 proposal that allocates at least* h *to at least one responder passes immediately.*

Thus what remains to be analyzed is voting on proposals that allocate *strictly less than* h *to both* responders.

4.3.1 Continuation equilibria when $x_{i1} < h$ for $i = 1, 2$

The following *algorithm* can be used to fully characterize the set of continuation equilibria following any such proposal. The idea behind this algorithm is to begin by positing a pattern of acceptance probabilities and then to back out the set of first round proposals that are consistent with that pattern.

Algorithm to characterize continuation equilibria following Period 1 proposals with $x_{i1} < h$ for $i = 1, 2$ (majority rule)

Let $\mathscr{M}$ be the set of *acceptance probability vectors* $\mu = \left(\mu_1^L, \mu_1^H, \mu_2^L, \mu_2^H\right)$ that are consistent with Corollary 1. For each $\mu \in \mathscr{M}$,

1. Identify the *belief vector* $\omega = \left(\omega_1^N, \omega_2^N\right)$ induced by a history in which both responders voted N. (Note that this is the only history leading to continuation play.)

2. Characterize the (set of) *continuation value vectors* $u = (u_1^L, u_1^H, u_2^L, u_2^H)$ that are consistent with Period 2 equilibrium play at a history that induces beliefs ω. Call this set $\mathscr{U}(\mu)$. (This will contain a unique vector unless the Proposer is indifferent between multiple proposals at ω.)

3. Identify the (set of) *Period 1 proposals* (x_1, x_2) with the property that there is *some* vector $u \in \mathscr{U}(\mu)$ such that

- if $\mu_i^T = 0$, then $x_i \leq (1 - \delta) b_i + \delta u_i^T$,
- if $\mu_i^T = 1$, then $x_i \geq (1 - \delta) b_i + \delta u_i^T$, and
- if $\mu_i^T \in (0, 1)$, then $x_i = (1 - \delta) b_i + \delta u_i^T$.

Then the set of Period 1 proposals identified in step (3) have the property that, following any proposal in that set, there exists a continuation equilibrium involving the Period 1 acceptance probability vector μ (as well as continuation values that support the equilibrium).

Despite its conceptual simplicity, the application of this algorithm to all $\mu \in \mathscr{M}$ involves quite a bit of rather tedious - essentially mechanical - work. We relegate these details to the Appendix and present only one example in the main text, followed by a geometric illustration of the final result.

Example

Fix $\mu = (1, 0, 1, 0)$. (Both responders vote Y if and only if they are low types.)

1. Beliefs following proposal failure (both vote N) are given by $\omega = (1, 1)$. (The Proposer is sure that both are high types.)

2. The Period 2 proposal can be any mix of $(h, 0)$ and $(0, h)$ (see Figure 1). Therefore, the set of possible continuation value vectors $u = (u_1^L, u_1^H, u_2^L, u_2^H)$ is

$$\mathscr{U}(\mu) = \{(qh, qh, (1-q)\,h, (1-q)h) : q \in [0, 1]\}$$

where q denotes the probability assigned to the Period 2 proposal $(h, 0)$.

3. Given $\left(\mu_i^L, \mu_i^H\right) = (1, 0)$, the proposal (x_1, x_2) must be such that there exists $q \in [0, 1]$ such that

$$\begin{aligned} (1-\delta)\,l + \delta q h \leq \quad & x_1 \quad \leq (1-\delta)\,h + \delta q h \\ (1-\delta)\,l + \delta\,(1-q)\,h \leq \quad & x_2 \quad \leq (1-\delta)\,h + \delta\,(1-q)\,h. \end{aligned}$$

Step (3) yields a condition identifying the set of Period 1 proposals (x_1, x_2) such that there exists a continuation equilibrium involving the acceptance probability vector $\mu = (1, 0, 1, 0)$. Geometrically, that area $A(\mu)$ is the convex hull of six points, obtained by substituting $q = 0$ and $q = 1$ into the lower and upper bounds of the intervals identified in step (3):

$$A(1,0,1,0) = conv \left\{ \begin{array}{cc} ((1-\delta)\,l, l + \delta\tau)\,, & (l + \delta\tau, (1-\delta)\,l)\,, \\ (h, (1-\delta)\,l)\,, & ((1-\delta)\,l, h)\,, \\ ((1-\delta)\,h, h)\,, & (h, (1-\delta)\,h) \end{array} \right\}.$$

At different points within this area, the continuation equilibria involve P mixing between $(h, 0)$ and $(0, h)$ in specific ways in case Period 2 is reached. Details follow from the above inequalities. (In all cases, P's expected Period 2 utility is $1 - h$.)

We can repeat this process for each $\mu \in \mathscr{M}$, identifying in each case a region in the $x_1 - x_2$ proposal space, such that for every proposal (x_1, x_2) in that region, there exists a continuation equilibrium involving the acceptance probability vector μ. Details are presented in the Appendix. As that analysis shows, the areas associated with different μ *overlap*, meaning that many Period 1 proposals are consistent with multiple continuation equilibria.

4.3.2 Proposer-preferred continuation equilibria

Given the multiplicity of continuation equilibria following first round proposals, we suggest that it makes sense to select, for each possible proposal, the continuation equilibrium associated with the largest expected payoff for the Proposer. This can be justified by imagining that P announces which proposal she intends to make in Period 2 in case it is reached. If such an announcement is consistent with a continuation equilibrium, it is credible and the associated expectations are self fulfilling. The following Lemma implies that this is equivalent to selecting the continuation equilibrium associated with the largest probability of passage.

Lemma 4. *Fix an arbitrary Period 1 proposal* (x_1, x_2)*. Suppose there exist two continuation equilibria with acceptance probability vectors* $\hat{\mu} \neq \tilde{\mu}$*, where* $\hat{\mu}$ *implies a higher probability of passage. Then P prefers the continuation equilibrium involving* $\hat{\mu}$*.*

Proof. See Appendix A.3. □

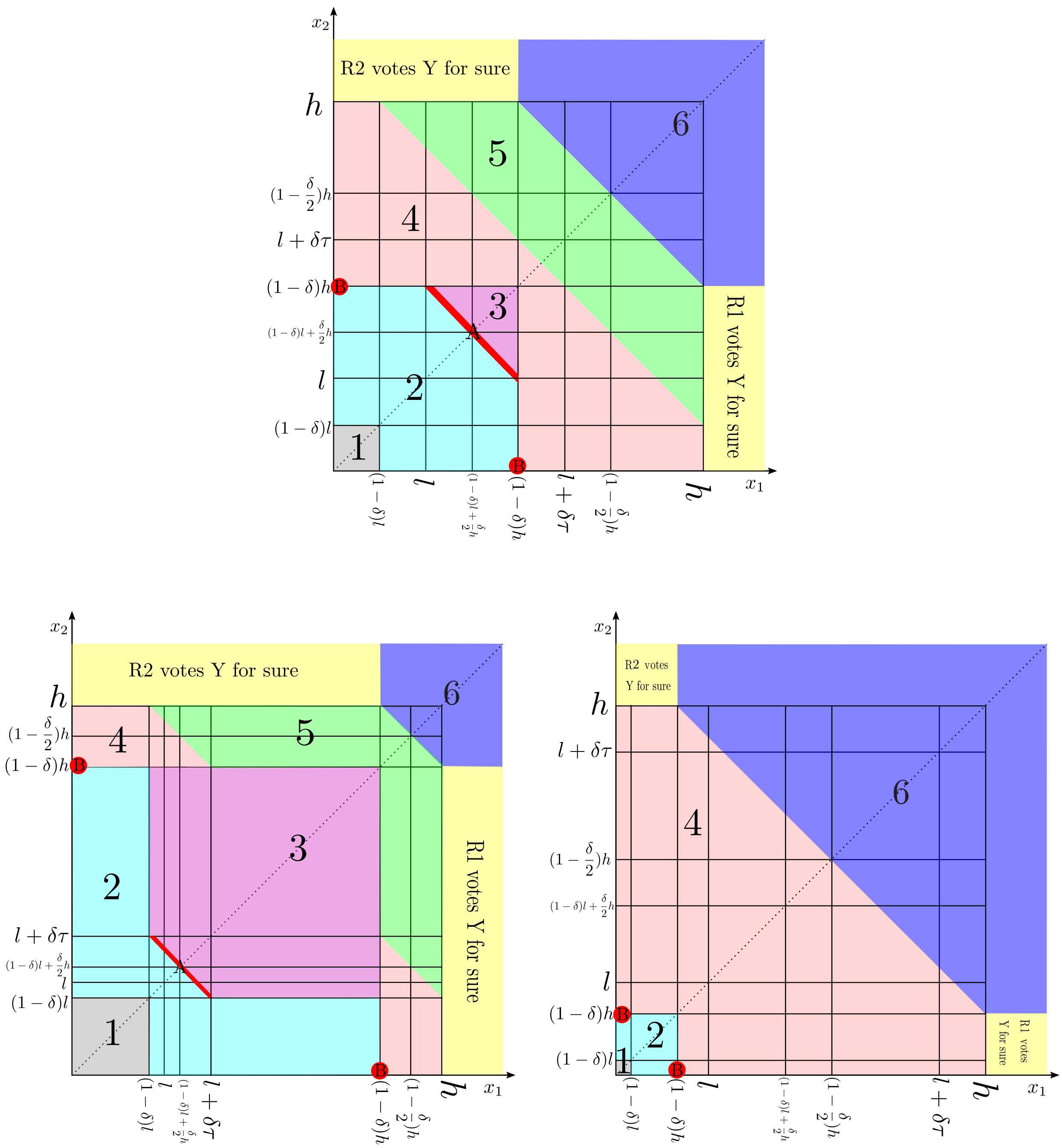
It turns out that this selection criterion eliminates all equilibria which involve mixing by either responder (see Appendix). Figure 2 presents the selected equilibria for the case that $\tau < \hat{\tau}^m$. Note that the shape of the areas depicted in this figure depend on the parameter values. For instance, see the lower two graphs of Figure 2 for variations in δ.

Proposition 1. *[Period 1 voting under majority rule] Following any Period 1 proposal* (x_1, x_2)*, the continuation equilibria involving the greatest probability of passage involve the following voting patterns (see Figure 2):*

1. *Region 1 (close to nothing to both responders): Both responders vote N for sure.*
2. *Region 2 (small shares to both): One responder votes Y if he is a low type.*
3. *Region 3 (moderate and similar shares to both): Each responder votes Y if he is a low type.*
4. *Region 4 (generous share to one / small share to other): One responder votes Y for sure.*
5. *Region 5 (generous share to one / moderate share to other): One responder votes Y for sure, the other votes Y if he is a low type.*
6. *Region 6 (generous shares to both): Both responders vote Y for sure.*

Regions with asymmetric voting probabilities consist of three sub-regions: When one responder's share is (sufficiently) larger, he is the more likely to vote Y. When offers are (sufficiently) similar, two equilibria with the same probability of passage coexist.

Figure 2: Equilibria following Stage 1 Proposal (majority rule)



The figure depicts the equilibria with the largest probability of passage following any Period 1 proposal x_1. The parameter values used are ($l = \frac{1}{10}$, $\tau = \frac{3}{10}$, and $\delta = \frac{1}{2}$) in the upper picture. In the lower pictures the situation with $\delta = \frac{1}{6}$ is depicted on the left and with $\delta = \frac{5}{6}$ on the right. See Proposition 1 for a description of each of the regions.

4.4 Period 1 proposal

Having characterized (Proposer-preferred) continuation equilibria following all Period 1 proposals, we turn to the P's Period 1 choice of proposal. Note that both the probability that a proposal *passes* as well as P's continuation value in case of failure are constant within each of the regions depicted in Figure 2. It follows that only proposals that lie at the south west corner or boundary of a given region can be optimal. Also note that proposals in regions 4, 5, and 6 pass for sure, and consequently regions 5 and 6 are dominated by the cheapest proposals in Region 4, labeled (B) in the figures.

It follows that P's best choice is one of the following four: Offer nothing to both responders (or any other proposal that fails for sure), offer either responder $(1-\delta)\,l$ and the other nothing (this passes with probability $\frac{1}{2}$), make any offer where $x_1 + x_2 = 2\,(1-\delta)\,l + \delta h$ and both get more than l (passes with probability $\frac{3}{4}$, marked (A) in Figure 2), or offer either responder $(1-\delta)\,h$ and the other nothing (which passes for sure, marked (B)). Comparing these options reveals that either (A) or (B) are optimal. The following Proposition identifies the conditions under which either is optimal.

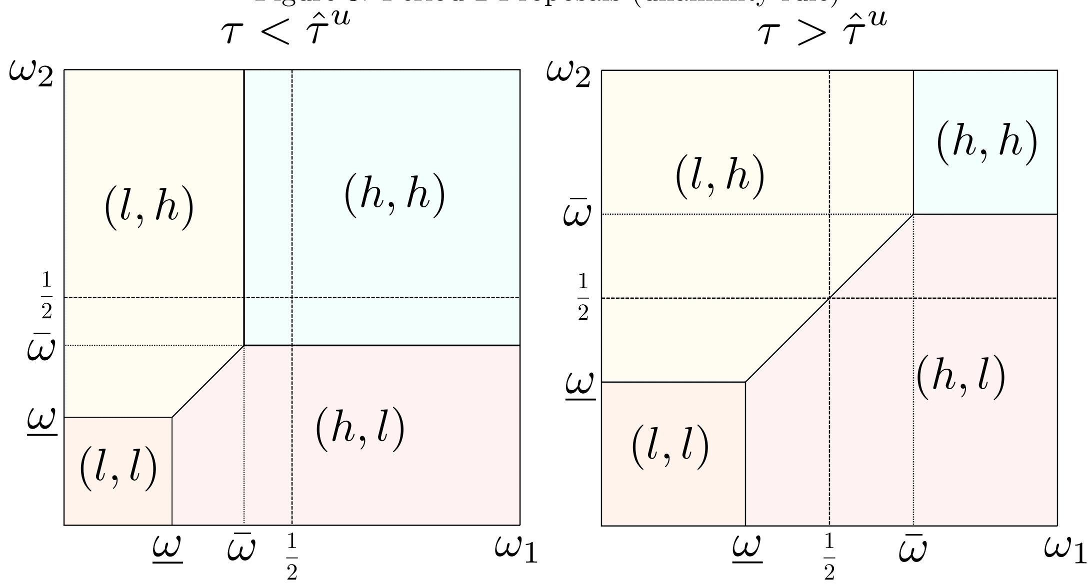
Proposition 2. *[Proposer-preferred equilibrium under majority rule] Consider majority rule and restrict attention to Proposer-preferred continuation equilibria following the first round proposal. Then,*

- ***If*** $\tau < \hat{\tau}^m$ **or** $\delta > \frac{4h-6l-1}{8h-6l-1}$***:*** *In Period 1, P offers* $(1-\delta)h$ *to one responder and zero to the other. The included player votes Y and so this proposal passes immediately. (If the included player were to vote N, P would offer h to the excluded responder in Period 2.)*

- ***Otherwise:*** *In Period 1, P proposes any* (x_1, x_2) *such that* $x_1 + x_2 = 2\,(1-\delta)\,l + \delta h$*,* $x_1 > (1-\delta)\,l$*, and* $x_2 > (1-\delta)\,l$*. Both players vote Y iff they are low and so this proposal passes with probability* $3/4$*. (If both responders vote N, P will offer one of them h in Period 2. The probability with which he includes R1 and R2 in this event varies depending on which responder is offered more in Period 1.)*

If δ is small, breakdown is likely and so responder prices depend on breakdown values. If in addition τ is large, the low type is significantly "cheaper" than the high type. Therefore, the "risky" option (A), where P makes a small offer to both responders, can be optimal. When δ is large, breakdown is unlikely, and so the fear of exclusion makes both types similarly cheap. If τ is small, the prices are similar anyway. Thus, in both cases, P is better off offering one responder enough to secure his vote for sure.

Figure 3: Period 2 Proposals (unanimity rule)



The figure depicts the Period 2 proposals that P prefers to make under unanimity rule given arbitrary beliefs about the responders' types. The horizontal and vertical axes measure the probability that R1 and R2 are of the high type, denoted ω_1 and ω_2 respectively. The left and right panels distinguish the cases where τ is smaller or greater than $\hat{\tau}^u$, which determines the optimal proposal in case $\omega_1 = \omega_2 = \frac{1}{2}$. We define $\bar{\omega} \equiv \frac{h-l}{1-h-l}$ and $\underline{\omega} \equiv \frac{h-l}{1-2l}$.

5 Unanimity rule

We now turn to an analysis of unanimity rule, following a similar outline based on backward induction, beginning with the Period 2 voting stage.

5.1 Period 2 voting and proposals

Lemma 1 applies to unanimity rule as well. Given this, the optimal Period 2 proposal is again determined by beliefs $\omega = (\omega_1, \omega_2)$. The formal analysis is presented in Appendix B.1. Figure 3 displays the result.

As under unanimity rule, the qualitative pattern of proposals depends on whether the difference between the low and high breakdown values exceeds a particular threshold, which we define as

$$\hat{\tau}^u \equiv \frac{1-2l}{3}.$$

The horizontal and vertical boundaries of the areas in these figures are defined by two levels of beliefs, which we denote $\bar{\omega} \equiv \frac{h-l}{1-h-l}$ and $\underline{\omega} \equiv \frac{h-l}{1-2l}$. The latter is always strictly smaller than $\frac{1}{2}$ (due to our assumption $2h < 1$). The former can be greater or less than $\frac{1}{2}$, thereby determining the optimal proposal at $\omega = \left(\frac{1}{2}, \frac{1}{2}\right)$, i.e., in case no information is revealed in Period 1. Specifically, $\bar{\omega} < \frac{1}{2}$ iff $\tau < \hat{\tau}^u$. Therefore, if $\tau < \hat{\tau}^u$, then P would offer (h,h) if Period 2 were reached without information being revealed, otherwise she would be indifferent between offering (h,l) or (l,h).

5.2 Continuation values and beliefs

As in the case of majority rule, the optimal Period 2 proposal and expected payoffs depend on beliefs ω induced by the prior history.[5] The following Lemma summarizes useful facts about these continuation values.

Lemma 5. *In any equilibrium under unanimity rule:*

- *At any history s, Responder i's Period 2 expected payoff satisfies*

$$l \leq u_i^L(s) \leq u_i^H(s) = h.$$

- *Let $\hat{s}$ and $\tilde{s}$ be two histories which induce beliefs $(\hat{\omega}_i, \hat{\omega}_{-i})$ and $(\tilde{\omega}_i, \tilde{\omega}_{-i})$, respectively. If $\hat{\omega}_i \geq \tilde{\omega}_i$ and $\hat{\omega}_{-i} = \tilde{\omega}_{-i}$, then $u_i^L(\hat{s}) \geq u_i^L(\tilde{s})$. That is, low types prefer to be perceived as high types.*

Proof. The first part follows from the fact that P will never offer either responder more than h, and both responders must vote Y for the proposal to pass. To see the second part, look at Figure 3, let $i = 1$, and hold ω_2 constant. If $\omega_2 < \underline{\omega}$ or $\omega_2 > \bar{\omega}$, increasing ω_1 only affects the Period 2 offer made to R1, possibly increasing it from l to h, clearly increasing his expected payoff. If $\omega_2 \in (\underline{\omega}, \bar{\omega})$, increasing ω_1 may cause the Proposer to shift from offering (l, h) to offering (h, l). Again, this can only increase R1's payoff, as he *may* end up with h and can never do worse than getting l. □

In contrast to majority rule, the set of circumstances under which Period 2 is reached is larger. In particular, it will be reached whenever at least one player has voted N. Therefore, in addition to (on path) beliefs about responders who voted N,

$$\omega_i^N = \frac{1 - \mu_i^H}{2 - \mu_i^H - \mu_i^L},$$

we must consider beliefs about players who voted Y, denoted

$$\omega_i^Y = \frac{\mu_i^H}{\mu_i^H + \mu_i^L}.$$

Recall that off-path beliefs are fixed by assumption to $\omega_i^N = 1$ if $\mu_i = (1, 1)$ and $\omega_i^Y = 0$ if $\mu_i = (0, 0)$.

[5] As above, some beliefs ω allow for P to mix between several proposals in Period 2, and so these ω are consistent with an entire *set* of proposals and expected payoffs. We will take this into account in the following analysis.

5.3 Period 1 voting

Fix an arbitrary Period 1 proposal (x_1, x_2). An immediate consequence of Lemma 5 is that a high type responder votes Y if and only if he is offered at least h.[6] This is in contrast to majority rule, where a high type responder is willing to accept $(1-\delta)h$ because he expects to be excluded if Period 2 is reached. Given this, low types must also vote Y if offered h or more.[7]

Lemma 6. *Let (x_1, x_2) be an arbitrary Period 1 proposal. Then in any equilibrium under unanimity rule,*

1. *if $x_i \geq h$, then $\mu_1^L = \mu_1^H = 1$, and*
2. *if $x_i < h$, then $\mu_i^H = 0$.*

Proof. See discussion above. □

It follows that a proposal that allocates at least h to both responders will pass immediately. Therefore, what remains to be analyzed is the probability with which low type responders vote Y on proposals that allocate strictly less than h to at least one responder. Within this set, it is helpful to consider separately proposals that allocate (at least) h to one responder and less than h to the other, before considering those which allocate strictly less than h to both.

Without loss of generality, consider a Period 1 proposal of the form (x_1, h) with $x_1 < h$. Then R2 votes Y for sure, making R1 pivotal. It is then straightforward to verify the following.

Lemma 7. *[Unanimity rule] Let (x_1, x_2) be a Period 1 proposal with $x_2 \geq h$ and $x_1 < h$. Then, in any equilibrium, R2 votes Y irrespective of his type and R1 votes Y only if he is a low type and*

$$x_1 \geq \begin{cases} l + \delta\tau & \text{if } \tau < \hat{\tau}^u \\ l + \frac{\delta}{2}\tau & \text{if } \tau > \hat{\tau}^u \end{cases}.$$

Proof. See discussion above. □

[6] Voting Y (N) on less (more) attractive proposals is weakly dominated. Voting Y when offered exactly h is without loss of generality.

[7] Suppose R1 votes N with positive probability on such a proposal (only) when his type is low. This is *strictly* worse than voting Y if R2 votes Y. (Instead of $x_1^1 \geq h$, R1 gets at most $(1-\delta)l + \delta h$). Suppose R2 votes N. Then the proposal fails irrespective of R1's vote. Given that he votes N only when he is a low type, beliefs will be $\omega_1^N = 0$. By voting Y instead, he would induce the belief $\omega_1^Y > \frac{1}{2}$, yielding a (weakly) greater continuation payoff. Thus, voting N is weakly dominated, a contradiction.

5.3.1 Continuation equilibria when $x_{i1} < h$ for $i = 1, 2$

Next, we turn to proposals that allocate *strictly less than h to both responders.* Then both responders vote N if they are high types, and what remains to be determined are the probabilities with which they accept when low. Denote these by $\mu^L = \left(\mu_1^L, \mu_2^L\right)$.

Mirroring our majority rule analysis, we now describe an *algorithm* that can be used to fully characterize the set of continuation equilibria following all first round proposals that allocate *strictly less than h to* both responders. Recall that, in contrast to majority rule, Period 2 can now be reached even in case one responder has voted Y in Period 1. Hence we denote Period 2 continuation values of a low type responder by $u_i^{V_i V_{-i}} = u_{i2}^L\left((x_1, x_2), (V_1, V_2)\right)$. For example, u_2^{NY} denotes the continuation value of the low type of R2 given Period 2 has been reached after a N-vote of R2 and a Y-vote of R1.

Algorithm to characterize continuation equilibria following Period 1 proposals allocating less than h to both responders (unanimity rule)

Let $\mathscr{M} = [0,1]^2$ be the set of low type *acceptance probability vectors* $\mu^L = \left(\mu_1^L, \mu_2^L\right)$. (Recall high types vote N on all proposals being considered.) For each $\mu^L \in \mathscr{M}$,

1. Identify the *belief vectors* $\omega^{NN} = \left(\omega_1^N, \omega_2^N\right)$, $\omega^{YN} = \left(\omega_1^Y, \omega_2^N\right)$ and $\omega^{NY} = \left(\omega_1^N, \omega_2^Y\right)$, induced by a history in which at least one responder has voted N, where in each case $\omega_i^N = \frac{1}{2-\mu_i^L}$ and $\omega_i^Y = 0$.

2. Characterize the (set of) *continuation value vectors* for low types $u_2^L = \left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right)$, where for example $\left(u_1^{YN}, u_2^{NY}\right)$ is consistent with Period 2 equilibrium play given beliefs ω^{YN} (after R1 voted Y and R2 voted N). If any of the belief vectors ω^{NN}, ω^{YN}, or ω^{NY} allows for multiple Period 2 equilibria, there will exist a *set* of consistent continuation value vectors, which we denote by $\mathscr{U}(\mu)$.

3. Identify the (set of) *Period 1 proposals* (x_1, x_2) with the property that there is *some* continuation value vector $u_2^L \in \mathscr{U}(\mu)$ such that:

- if $\mu_i^L = 0$,

$$\frac{\mu_{-i}^L}{2} x_i + \left(1 - \frac{\mu_{-i}^L}{2}\right)\left[(1-\delta)\, l + \delta u_i^{YN}\right] \le (1-\delta)\, l + \delta \left[\frac{\mu_{-i}^L}{2} u_i^{NY} + \left(1 - \frac{\mu_{-i}^L}{2}\right) u_i^{NN}\right],$$

  where $\frac{\mu_{-i}^L}{2}$ is the probability with which the "other" responder votes Y.

- if $\mu_i^L = 1$, the inequality is reversed, and
- if $\mu_i^L \in (0,1)$, it holds with equality.

This condition can be conveniently restated as saying that i votes Y as a low type if $\mu_{-i}^L \neq 0$ and

$$x_i \ge (1-\delta)\, l + \delta \left[u_i^{NY} + \frac{2 - \mu_{-i}^L}{\mu_{-i}^L}\left(u_i^{NN} - u_i^{YN}\right)\right]$$

with mixing permitted in case of equality.

We illustrate the application of the algorithm using an example. All other cases are analyzed in the Appendix.

Example: Fix $\mu^L = (1,1)$.

That is, both responders vote Y if and only if they are low types. Any responder voting N reveals that he is a high type. Thus,

1. Period 2 beliefs are given by $\omega^{NN} = (1,1)$, $\omega^{YN} = (0,1)$ and $\omega^{NY} = (1,0)$.

2. The continuation value vector is uniquely determined as

$$\left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (h, h, l, h, h, l)\,.$$

3. Given $\mu_i^L = 1$, the relevant set of Period 1 proposals consists of those satisfying

$$x_i \geq l + 2\delta(h-l) \text{ for } i = 1, 2$$

which satisfies $x_i < h$ only if $\delta < \frac{1}{2}$.

Step (3) gives a condition identifying the set of Period 1 proposals x such that there exists a continuation equilibrium involving the acceptance probability vector $\mu = (1,0,1,0)$. Geometrically, this set is the following convex hull

$$A(1,0,1,0) = conv\left\{(h,h), (l+2\delta\tau, l+2\delta\tau), (l+2\delta\tau, h), (h, l+2\delta\tau)\right\}.$$

We can repeat this process for each $\mu^L \in \mathscr{M}$, resulting in a full characterization of all continuation equilibria following any Period 1 proposal. Each $\mu^L \in \mathscr{M}$ is associated with a region in proposal space such that for each Period 1 proposal in that region, a continuation equilibrium involving the acceptance probability vector $\mu = (\mu_1^L, 0, \mu_2^L, 0)$ exists. The details of this analysis are presented in the Appendix.

5.3.2 Continuation equilibria with the largest probability of passage

As in the case of majority rule, the regions associated with different continuation equilibria overlap, reflecting the multiplicity of equilibria that results from the endogenous signaling effects associated with each voting pattern. We again resolve this multiplicity by selecting the equilibrium associated with the largest probability of passage.[8] The results are summarized in Proposition 3. Figure 4

[8] As in the case of majority rule, this assumption can be motivated by imagining that P could make a self-fulfilling announcement as to her intentions. The argument is complicated slightly by the fact that, under unanimity rule, P may prefer equilibria associated with a lower probability of passage following certain Period 1 proposals. However, it can be shown that such proposals are dominated by others, so that our selection criterion is without consequence for equilibrium play. See Section B.3 in Appendix for a more detailed discussion.

provides a diagrammatic illustration for the case that $\tau > \hat{\tau}^u$ and $\delta < \frac{\bar{\mu}}{2}$.

Proposition 3. *[Period 1 voting under unanimity rule] Assume unanimity rule. Following any Period 1 proposal (x_1, x_2), the continuation equilibria involving the greatest probability of passage involve the following voting patterns (see Figure 4):*

1. *Region 1 (allocating a small share to at least one responder): Both responders vote N irrespective of their type.*
2. *Region 2 (allocating a small share to one responder and at least h to the other): The responder offered less votes N, the other votes Y, both irrespective of their type.*
3. *Region 3 (close to symmetric, allocating moderate shares to both): Each responder votes Y with probability $\bar{\mu} = \frac{3h-l-1}{h-l}$ if he is a low type.*
4. *Region 4 (less symmetric, allocating a moderate share to one and a more generous share to the other): The responder offered less votes Y with probability $\bar{\mu}$ if he is a low type. The other votes Y with a larger probability if he is a low type.*
5. *Region 5 (even less symmetric proposals, allocating a moderate share to one and a generous share to the other): The responder offered less votes Y with probability $\bar{\mu}$ if he is a low type. The other votes Y for sure if he is a low type.*
6. *Region 6 (allocating more than $l + 2\delta\tau$ to each responder): Each responder votes Y if he is a low type.*
7. *Region 7 (allocating a moderate share to one responder and at least h to the other): The responder offered less votes Y if he is a low type, the other votes Y irrespective of his type.*
8. *Region 8 (allocating at least h to both responders): Both responders vote Y irrespective of their type.*

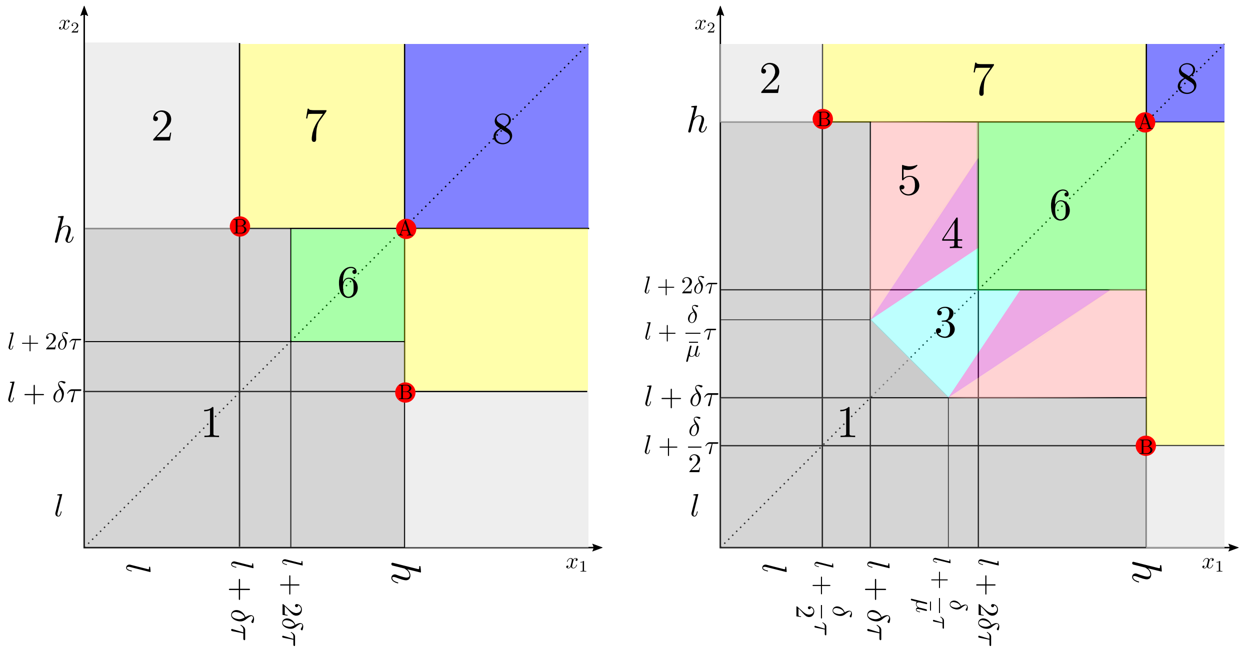
These equilibrium areas are ordered in ascending order of the probability of passage.

5.4 Period 1 proposal

Having characterized the continuation equilibria with the largest probability of passage following all Period 1 proposals, we turn to P's Period 1 choice of proposal. The probability that a proposal *passes* is constant within all the areas depicted in Figure 4, with Region 4 being an exception.[9] For

[9]Region 4 involves both responders mixing between voting Y and N. One responder votes Y with fixed probability $\bar{\mu}$, the other's probability varies between $(\bar{\mu}, 1)$.

Figure 4: Equilibria following Stage 1 Proposal (unanimity rule)



The figure depicts the equilibria with the largest probability of passage following any Period 1 proposal x_1. The parameter values used are ($l = \frac{1}{10}$, $\tau = \frac{2}{10}$, $\delta = \frac{1}{4}$ so that $\tau < \hat{\tau}^u$) in the picture on the left and ($l = \frac{1}{10}$, $\tau = \frac{1}{3}$, $\delta = \frac{1}{4}$ so that $\tau > \hat{\tau}^u$ and $\bar{\mu} = \frac{3}{5}$) in the one on the right.

these areas, it follows that only proposals that lie at the southwest boundary or corner of a given area can be optimal. It can be shown that the best offer in Region 5 dominates all offers in Region 3 and Region 4, respectively.

Thus P's optimal Period 1 proposal must be one of the following: A proposal that fails for sure (Region 1 or 2); a proposal such that one responder votes Y with probability $\bar{\mu}$ as a low type while the other votes Y for sure as a low type (Region 5); a proposal which both responders support if and only if they are low (Region 6); buying both responders for sure (point (**A**)); or point **(B)** where one responder is paid enough to secure his vote while the other is offered a smaller share such that he votes Y if and only if he is low. If $\tau < \hat{\tau}^u$, it can be shown that option **(A)** dominates all other options. If $\tau > \hat{\tau}^u$, it can be shown that proposal **(B)** dominates all other proposals.

Proposition 4. *[Proposer-preferred equilibrium under unanimity rule] Consider unanimity rule and restrict attention to the continuation equilibria with the largest probability of passage following the first round proposal (see Figure 4). Then,*

- **If $\tau < \hat{\tau}^u$:** In Period 1, P proposes (h, h) and both responders vote Y. (If either responder votes N, P proposes (h, h) again in Period 2.)

- **If $\tau > \hat{\tau}^u$:** In Period 1, P offers h to one responder, who votes Y for sure, and $l + \frac{\delta}{2}\tau$ to the other, who votes Y iff he is low. (If the responder that was offered the smaller share votes N, he is offered h in Period 2, while the other responder is offered l.)

The intuition underlying this result is as follows. If the difference between high and low breakdown values, τ, is small, P prefers to pay both responders h to secure immediate agreement. For larger τ, P prefers to take a risk by trying to buy one responder "cheap", securing agreement with probability $\frac{1}{2}$. Even so, P offers the other responder h in order to secure his vote for sure. Doing so not only increases the probability of passage from $\frac{1}{4}$ to $\frac{1}{2}$, it also makes the disadvantaged responder cheaper because the price for his vote decreases from $\max\{l + 2\delta\tau, h\}$ to $l + \frac{\delta}{2}\tau$ if he expects his counterpart to vote Y for sure rather than only when he is a low type. The reason is that the disadvantaged responder becomes pivotal, eliminating the negative signaling value of voting Y (as this results in immediate agreement).[10]

6 Unanimity vs. Majority Rule

We now turn to a comparison of unanimity and majority rule. Our goal is to make formally precise the notion that responders are "more expensive" and that immediate agreement is "less likely" or "more difficult to achieve" under unanimity rule. Such a comparison is complicated by the fact that the "price" of a responder's vote depends on several factors, including his expectations concerning the other responder's voting behavior. Another complication is that some offers can induce responders to mix between voting Y and N. In what follows, we use the term "price" to refer to the amount that a responder must be offered in order to secure his vote, i.e., in order for him to vote Y with probability 1.

6.1 Period 1 "prices"

Propositions 1 and 3 identify conditions on x_{i1} such that a low or high type responder i would vote Y on a proposal in equilibrium, taking the other responder's voting probabilities as given. For simplicity, we restrict attention to cases where μ_{-i} reflects a pure strategy, i.e., $\mu_{-i} = \left(\mu_{-i}^L, \mu_{-i}^H\right) \in \{(0,0),(1,0),(1,1)\}$. We define the corresponding "prices" as follows.

[10] To understand why the threshold determining the equilibrium is exactly $\hat{\tau}^u$, consider a Period 1 offer such that R2 votes Y for sure. Then, R1 anticipates that voting N (inducing a belief $(\omega_1^N, \frac{1}{2})$ with $\omega_1^N > \frac{1}{2}$) leads to second period proposal (h,h) or (h,l) depending on whether $\tau \lesseqgtr \hat{\tau}^u$, causing a discreet change in his "price" at that point. One might expect that P would be more willing to "take a risk" in Period 1 if δ is larger. But the same is true for the responder, who will perceive a greater signaling value from voting N and so his price goes up. It turns out that these factors cancel so as to make δ irrelevant.

Definition 2. Let $\mu_{-i} \in \{(0,0),(1,0),(1,1)\}$ be fixed. Then the Period 1 price of a type T responder under voting rule $v \in \{m,u\}$ is denoted $z_T^v(\mu_{-i})$, and defined as the smallest value of x_{i1} such that $\mu_i^T = 1$ is consistent with the relevant Proposition 1 or 3.

Under majority rule, each type's price is actually independent of μ_{-i}, and given by $(1-\delta)b_i$. That is, $z_H^m(\mu_{-i}) = (1-\delta)h$ and $z_L^m(\mu_{-i}) = (1-\delta)l$. These prices reflect the fact that P can credibly announce that she plans to exclude the responder to whom she is making a Period 1 offer if that responder votes N. When $\delta \to 1$, this threat of exclusion looms large and the price of a vote tends to zero. As $\delta \to 0$, the responder's breakdown value effectively becomes an outside option and so the price approaches b_i.

Under unanimity rule, the high type responder's price is also independent of μ_{-i} and given by $z_H^u(\mu_{-i}) = h$ (see Lemma 6). Thus, high types are unambiguously more expensive under unanimity rule than under majority rule. This reflects the fact that both responders have veto power, so there is no reason to accept anything less than h.

In contrast, a *low type* responder's price under unanimity rule does depend on μ_{-i}. In particular,

$$z_L^u(1,1) = \begin{cases} l + \frac{\delta}{2}\tau & \tau > \hat{\tau}^u \\ l + \delta\tau & \tau < \hat{\tau}^u, \end{cases}$$

$$z_L^u(1,0) = min\Big\{l + 2\delta\tau, h\Big\},$$

and

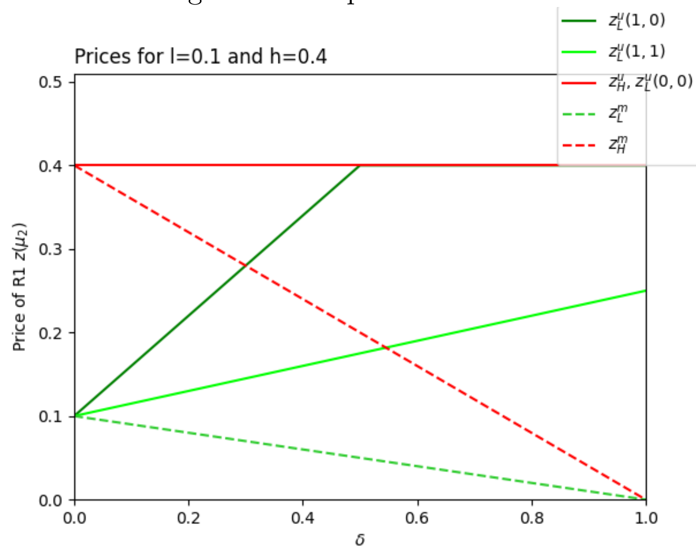
$$z_L^u(0,0) = h.$$

Note that each of these prices exceeds l and so the low type is also unambiguously more expensive under unanimity rule than under majority rule. This reflects two circumstances. First, veto power implies that a payoff of at least l can be guaranteed by simply voting N consistently. Second, the low type can hope to achieve an even greater payoff if P can be made to believe that he is a high type. Thus, the low type must be compensated for revealing his type. This *signaling incentive* is more relevant the lower is the probability with which the other responder votes Y. When $\mu_{-i} = (1,1)$, Responder i expects to be pivotal and is "cheaper" than when $\mu_{-i} = (1,0)$, in which case he expects that Period 2 might be reached even if he votes Y, in which case he would be "punished" for revealing his type. When $\mu_{-i} = (0,0)$, this signaling incentive is all that is relevant, and so the low type must pool with the high type, inducing a "price" of h.[11]

Figure 5 presents responder prices as functions of δ for a given combination of parameter values

[11] Under unanimity rule, there are also cutoff values above which low type responders mix between voting Y and

Figure 5: Responders' Prices



R1's prices as a function of the continuation probability δ under unanimity rule (solid lines) and majority rule (dashed lines) given R2's voting probabilities and given that $h = \frac{4}{10}$ and $l = \frac{1}{10}$ (i.e., $\tau > \hat{\tau}^u$). R2 is expected to play a pure strategy. For instance, $z_L^u(1,0)$ is the price of the low type of R1 under unanimity rule given that R2 votes Y if and only if he is of a low type.

($l = \frac{1}{10}$ and $\tau = \frac{3}{10}$). As we have noted, majority rule prices (*dashed lines*) are decreasing in δ, as the threat of exclusion looms large when breakdown is unlikely. In contrast, unanimity rule prices (*solid lines*) *increase* in δ. This reflects the fact that the *positive signaling value* of voting N is more important the more likely it is that P will be able to make a second proposal.

6.2 Probability of Breakdown

In order to assess the efficiency of the two rules, we compare the probability of delay and breakdown. Under unanimity rule, delay can occur if and only if the two responder types are sufficiently different ($\tau > \hat{\tau}^u$). Then, P offers h to one and to the other something that they accept only if they are low. If the latter rejects, P offers him h in Period 2, and l to the first. In both cases, the disadvantaged responder accepts with probability $\frac{1}{2}$. Therefore, delay occurs with probability $\frac{1}{2}$ in both periods, and the probability of inefficient breakdown is $\frac{1}{2}\left(1 - \delta + \frac{\delta}{2}\right) = \frac{1}{2} - \frac{\delta}{4}$. If, on the other hand, the types are similar ($\tau < \hat{\tau}^u$), P prefers to offer both responders h in Period 1 and agreement is certain.

Under majority rule, delay can occur if and only if $\tau > \hat{\tau}^m$ and $\delta < \frac{4h-6l-1}{8h-6l-1}$. That is, types are sufficiently different and the probability of breakdown is sufficiently large. (Only) under these

N. These cutoff values are $\begin{cases} l + \frac{\delta}{2}\tau & \mu_{-i} = (1,1) \\ l + \delta\tau & \mu_{-i} = (1,0) \wedge \tau > \hat{\tau}^u \\ l + 2\delta\tau & \mu_{-i} = (1,0) \wedge \tau < \hat{\tau}^u \end{cases}$. Again, each of these "prices" is larger than l and so unambiguously larger than the low type's price under majority rule.

conditions, high types will be significantly more expensive than low types, so that P prefers to "gamble" by offering both responders something that only low types accept. Thus, delay occurs with probability $\frac{1}{4}$ (if both responders are high types). If the second period is reached, agreement is certain. Thus, the probability of *breakdown* is $\frac{1-\delta}{4}$. For $\tau < \hat{\tau}^m$ or $\delta > \frac{4h-6l-1}{8h-6l-1}$, agreement in Period 1 is certain under majority rule.

In sum, the probability of a breakdown is larger under unanimity rule under all conditions except when $\tau \in (\hat{\tau}^m, \hat{\tau}^u)$ (nonempty if $l < \frac{1}{14} \approx 0.07$) and δ is close enough to 0.[12] This result confirms the central part of the informal argument that motivates our analysis, namely that inefficiencies due to "tough" bargaining are more likely to occur under unanimity rule than under majority rule.

It is perhaps not surprising that agreement is more difficult to achieve under unanimity rule, given that P must convince both responders rather than just one. But our analysis reveals an additional, less obvious point: This is that part of the reason for possible inefficiencies is that unanimity rule causes responders to become more "expensive", possibly inducing the Proposer to take a risk by making a proposal that could fail. Under majority rule, in contrast, even high type responders will be "cheap" to buy (except under special circumstances) such that P can afford to make a proposal that will pass for sure.

The generally lower probability of (inefficient) breakdown under majority rule, in combination with the mostly lower prices for responders, lead to the Proposer being better off under majority rule than under unanimity rule. Responders in contrast prefer, in most cases, unanimity rule. While responders can be sure to end up with at least their breakdown value under unanimity rule, they must fear exclusion under majority rule. Hence, high types certainly prefer unanimity voting. Low types, in contrast, prefer majority rule if $\tau \in (\hat{\tau}^u, \hat{\tau}^m)$ and $\delta < \frac{3h-l}{h-3l}$ (which is non-empty if $l \in \left(\frac{1}{14}, \frac{1}{6}\right)$). Here, the responder, under unanimity rule, fears breakdown whereas under majority rule agreement is certain (due to $\tau \in (\hat{\tau}^u, \hat{\tau}^m)$). In addition, the fear of Period 2 exclusion under majority rule is low (due to the low δ) so that P ends up making a large Period 1 offer.

[12] These conditions are extremely restrictive. For instance, if $l = 0.05$, then we must have $\tau \in (0.275, 0.3)$ and $\delta \leq 0.0\bar{6}$ (if $\tau \approx 0.3$). To get an intuition for this, consider the extreme case of $\delta = 0$ and $l = 0$. Under both rules, P compares two types of proposals: Those that pass for sure because she offers both responders enough to secure even the high type's vote, and those where she "gambles" by offering at least one responder something he might reject. If $\tau > \frac{1}{4}$, P prefers to take the gamble under majority rule and offer both responders the low type's price $l = 0$, which passes with probability $\frac{3}{4}$ (i.e., unless both are high types). Under unanimity rule, however, a similar gamble is too risky if $\tau < \frac{3}{4}$. P needs both responders to vote Y. Thus, even if she offers the low type's price to only one of the responders (and buys the other for sure), there is only a 50% chance that the proposal will pass. Therefore, P prefers to buy both high types under unanimity rule. Thus for these special parameter conditions, delay can only occur under majority rule.

7 Discussion

The choice between alternative q-majority rules, including unanimity rule, is a central problem in constitutional design. Unanimity rule has the attractive property that decisions taken are guaranteed to be *efficient* in the sense that all members prefer the proposed change relative to the status quo. A potential disadvantage is that agreement might be more difficult to reach under unanimity rule *even if it is efficient*. This is because, even when all members of a group can potentially benefit, each individual member may have an incentive to overstate their opposition, in an attempt to secure a larger share of the surplus that results from agreement. Arguments along these lines have been made informally, among others by Buchanan and Tullock (1965). The goal of the present paper was to provide an explicit formalization of this argument in a multilateral bargaining game with private information.

Our central results confirm the informal argument that unanimity rule creates incentives to "act tough" and thereby makes agreement more difficult to achieve. In general, the prices necessary to secure a responder's vote are higher under unanimity rule than under majority rule, ceteris paribus. The main reason for this is that voting N carries a positive "signaling value" under unanimity rule, while it is discouraged through the threat of subsequent exclusion under majority rule. The combination of higher prices and the need to secure more votes implies that the proposer will more often choose to risk delay and breakdown under unanimity rule, resulting in a larger probability of inefficient disagreement. In addition to confirming the informal argument outlined above, the formal analysis yields additional insights into the underlying incentives and the properties of optimal strategies.

For example, the analysis yields interesting comparative statics with respect to the continuation probability δ. Under unanimity rule, the high type's price always corresponds to his breakdown value, since he can veto any other outcome. The low type's price, however, is larger than his breakdown value and increases in δ. This is because acting "tough" is both less costly and more beneficial if the next period is reached with a larger probability following proposal failure. Under majority rule, in contrast, responders' prices are generally lower than their breakdown values and decreasing in δ, and indeed approach zero as δ approaches 1. The intuition is that acting "tough" becomes counterproductive if Period 2 is likely to be reached, as the "tough" responder will simply be excluded from the Period 2 coalition.

Another insight concerns strategic complementarities in responders' voting choices. Under unanimity rule, a responder is more willing to vote Y (in the sense that his price is lower) the higher the probability that the *other* responder votes Y, in which case his own vote becomes pivotal. The reason is that voting Y carries a *negative signaling value* in case the proposal fails. The more likely it is that the other responder votes Y, the less relevant this becomes.[13] This translates into an incentive for the proposer to offer one responder enough to secure his vote for sure, thereby making the other responder pivotal and consequently "cheap".[14] Interestingly, the opposite is true under majority rule. Here, a responder is *less* willing to agree (his price is higher) the more likely the other responder is to vote Y.[15]

Finally, our analysis will be of interest to researchers working on formal approaches to bargaining and voting with private information. We were surprised by the the rich complexity that we encountered in analyzing a model that might appear quite simple at first sight. (Three players, a fixed proposer, only two types, and two rounds of negotiation.) It would be interesting to extend the analysis to more players, types, and longer / infinite time horizon. However, this is beyond the scope of our paper, and indeed our analysis might serve as caution that such a model may become intractable.

[13] There is one exception to this pattern: For a small interval of τ, R1's price is lower if R2 mixes with a certain probability μ_2^L when low than if R2 votes Y for sure when low. The reason is that if R2 votes Y if low, then in Period 2 after (N,N) P will offer the high share to both responders. This makes voting N more attractive compared to the case where P offers the high share only to R1 (and thus the Period 2 proposal only passes if R2 is low) - as in the case with $\mu_2^L < \bar{\mu}$. For this to hold we need $\mu_2^L \in (\frac{1}{2}, \bar{\mu})$ and thus $\tau > \frac{2-4l}{5}$ (so that $\bar{\mu} > \frac{1}{2}$).

[14] In a model involving a larger number of players, this may translate into an interesting trade-off: P will wish to offer a sufficiently large number of responders large enough shares to secure their certain agreement such that the remaining responders are sufficiently convinced that agreement is likely so as to make their votes pivotal, and therefore "cheap".

[15] The reasons underlying this effect are subtle and have to do with the beliefs formed in case the first round proposal fails. There, the precise way this plays out is specific to the three player case. The general insight is that under majority rule, responders are more likely to accept if they believe that they will be excluded from a future coalition. This threat of exclusion, in turn, will be more credible if *other* responders are not induced to signal their types, i.e., if P offers them something such that they vote either Y or N irrespective of their type. It appears potentially interesting to investigate in more detail the implications of this for the design of optimal proposals in larger groups.

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A Majority rule

A.1 Period 2 proposal (majority rule)

Consider any history preceding the Period 2 proposal stage, and let the corresponding beliefs be given by (ω_1, ω_2). Suppose without loss of generality that $\omega_1 \leq \omega_2$. By Lemma 1, $x_2 = (h, 0)$ and $x_2 = (0, h)$ pass for sure, both yielding a Proposer payoff of $1 - h$. $x_2 = (l, l)$ passes with probability $(1 - \omega_1\omega_2)$, yielding $(1 - \omega_1\omega_2)(1 - 2l)$. $x_2 = (l, 0)$ dominates $x_2 = (0, l)$, and passes with probability $(1 - \omega_1)$, yielding $(1 - \omega_1)(1 - l)$. Comparing these options establishes that the optimal Period 2 proposal is given by

$$x_2(\omega_1, \omega_2) = \begin{cases} (h, 0) \text{ or } (0, h) & \text{if } \omega_1 > \frac{h-l}{1-l} \text{ and } \omega_1\omega_2 > \frac{h-2l}{1-2l} \\ (l, 0) & \text{if } \omega_1 < \frac{h-l}{1-l} \text{ and } \frac{1-\omega_1}{1-\omega_1\omega_2} > \frac{1-2l}{1-l} \\ (l, l) & \text{if } \omega_1\omega_2 < \frac{h-2l}{1-2l} \text{ and } \frac{1-\omega_1}{1-\omega_1\omega_2} < \frac{1-2l}{1-l} \end{cases}$$

It is informative to consider the optimal proposal for $\omega = \left(\frac{1}{2}, \frac{1}{2}\right)$, i.e., following any history in which no information has been revealed:

$$x_2\left(\frac{1}{2}, \frac{1}{2}\right) = \begin{cases} (h, 0) \text{ or } (0, h) & \text{if } \tau < \frac{1+2l}{4} \\ (l, l) & \text{otherwise} \end{cases}$$

A.2 Period 1 voting (majority rule)

We use the algorithm described in the main text to characterize the (set of) continuation equilibria following arbitrary Period 1 proposals that allocate less than h to both responders. The algorithm involves identifying, for each acceptance probability vector μ consistent with Lemma 3, the set of Period 1 offers x_1 such that a continuation equilibrium involving the acceptance probabilities μ exists. The set of offers for which this is true will typically be characterized by a set of inequalities. In some cases, these inequalities will involve parameters reflecting alternative mixing probabilities for offers made in Period 2. The set of proposals x_1 which can satisfy the inequalities for some set of feasible parameters constitutes an area in Period 1 proposal space, which we will denote by $A(\mu)$. As we will see, $A(\mu)$ will typically be the convex hull of a set of points obtained by identifying the largest and smallest values that these parameters can take.

Acceptance vector $\mu = (0, 0, 0, 0)$ Then (1) beliefs following failure are $\omega = \left(\frac{1}{2}, \frac{1}{2}\right)$. For step (2), we must make a case distinction:

(2a) If $\tau < \hat{\tau}^m$: The Period 2 proposal is any mix between $(h, 0)$ and $(0, h)$. Let ρ denote the probability that P proposes $(h, 0)$. Then the set of possible continuation values is $\mathscr{U}(\mu) = \{(\rho h, \rho h, (1-\rho)h, (1-\rho)h) : \rho \in [0,1]\}$. Then (3a) to support μ, there must exist $\rho \in [0,1]$ such that

$$\begin{aligned} x_1 &\leq (1-\delta)l + \delta\rho h \\ x_2 &\leq (1-\delta)l + \delta(1-\rho)h \end{aligned}$$

The area for which such a ρ exists can be characterized by first considering the extreme cases $\rho = 0$ and $\rho = 1$. The first implies that $x_1 \in [0, (1-\delta)l]$ and $x_2 \in [0, (1-\delta)l + \delta h]$. These conditions describe the convex hull of the four points $(0,0)$, $(0, (1-\delta)l + \delta h)$, $((1-\delta)l, 0)$, $((1-\delta)l, (1-\delta)l + \delta h))$. The second case ($\rho = 1$) is symmetric, requiring $x_1 \in [0, (1-\delta)l + \delta h]$ and $x_2 \in [0, (1-\delta)l]$, satisfied within the convex hull of the four points $(0,0)$, $((1-\delta)l + \delta h, 0)$, $(0, (1-\delta)l)$, $((1-\delta)l + \delta h), (1-\delta)l)$. Next, we can imagine varying ρ between 0 and 1, tracing out the convex hull off all points identified thus far. Note that $(0, (1-\delta)l)$ itself is in between $(0,0)$ and $(0, (1-\delta)l + \delta h)$, and similarly for $((1-\delta)l, 0)$. And the point $(0,0)$ appeared in both cases. Thus of the eight points identified, three are redundant. The area we are looking for is thus the convex hull of five points:

$$\begin{aligned} A(0,0,0,0) = conv\{&(0,0)\,, (0, (1-\delta)l + \delta h)\,, ((1-\delta)l, (1-\delta)l + \delta h)\,, \\ &((1-\delta)l + \delta h, (1-\delta)l)\,, ((1-\delta)l + \delta h, 0)\}. \end{aligned}$$

Similar reasoning can be applied to each of the cases we will consider in what follows, and we will not spell out all details each time.

(2b) If $\tau > \hat{\tau}^m$: The Period 2 proposal is (l, l). Therefore the continuation values are uniquely determined as

$$u = \left(l, \frac{l+h}{2}, l, \frac{l+h}{2}\right)$$

(Low types will vote Y in Period 2, high types will vote N and expect the other to vote Y with prob $\frac{1}{2}$.) Then (3b), to support μ, the proposal must satisfy

$$x_i \leq l \text{ for } i = 1, 2$$

and so the corresponding area is

$$A(0,0,0,0) = conv\left\{(0,0), (0,l), (l,l), (l,0)\right\}.$$

Acceptance vector $\mu = (1,0,0,0)$ Then (1) beliefs following Period 1 proposal failure are $\omega = \left(1, \frac{1}{2}\right)$. (2) The Period 2 proposal is any mix between $(h,0)$ and $(0,h)$. Therefore, $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. Then (3) to support μ, there must exist $q \in [0,1]$ such that

$$\begin{aligned}(1-\delta)l + \delta qh \leq \quad & x_1 \quad \leq (1-\delta)h + \delta qh \\ & x_2 \quad \leq (1-\delta)l + \delta(1-q)h.\end{aligned}$$

The corresponding area is

$$\begin{aligned}A(1,0,0,0) = conv\{&((1-\delta)l, 0), ((1-\delta)l, (1-\delta)l + \delta h), \\ &((1-\delta)h, (1-\delta)l + \delta h), (h, (1-\delta)l), (h,0)\}.\end{aligned}$$

Acceptance vector $\mu = (1,1,0,0)$ (1) Period 2 beliefs are $\omega = (1, \frac{1}{2})$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned}x_1 \quad & \geq (1-\delta)h + \delta qh \\ x_2 \quad & \leq (1-\delta)l + \delta(1-q)h.\end{aligned}$$

The corresponding area is

$$\begin{aligned}A(1,1,0,0) = conv\{&((1-\delta)h, 0), ((1-\delta)h, (1-\delta)l + \delta h), \\ &(1 - (1-\delta)\,l - \delta h, (1-\delta)\,l + \delta h), (1,0)\}.\end{aligned}$$

Acceptance vector $\mu = (1,0,1,1)$ (1) Beliefs following failure are $\omega = (1,1)$ (R2's N-vote is "unexpected"). Again (2) $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned}(1-\delta)l + \delta qh \leq \quad & x_1 \quad \leq (1-\delta)h + \delta qh \\ (1-\delta q)\,h \leq \quad & x_2.\end{aligned}$$

The corresponding area is

$$A(1,0,1,1) = conv\{((1-\delta)l, h), ((1-\delta)\, l, 1-(1-\delta)\, l), \\ (h, 1-h), (h, (1-\delta)h), ((1-\delta)l + \delta h, (1-\delta)h)\}.$$

Acceptance vector $\mu = (1,1,1,1)$: (1) Period 2 beliefs are $\omega = (1,1)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &\geq (1-\delta)h + \delta qh \\ x_2 &\geq (1-\delta q)\, h \end{aligned}$$

The corresponding area is

$$A(1,1,1,1) = conv\{((1-\delta)h, h), ((1-\delta)\, h, 1-(1-\delta)\, h), (1-(1-\delta)\, h, (1-\delta)\, h), (h, (1-\delta)\, h)\}.$$

Acceptance vector $\mu = (1,0,1,0)$: This was covered as an example in the main text.

Acceptance vectors that involve one responder mixing These are: $\mu = (\mu_1^L, 0, 0, 0)$, $\mu = (\mu_1^L, 0, 1, 1)$, $\mu = (\mu_1^L, 0, 1, 0)$ and $\mu = (1, \mu_1^H, 0, 0)$, $\mu = (1, \mu_1^H, 1, 1)$, $\mu = (1, \mu_1^H, 1, 0)$.

Acceptance vector $\mu = (\mu_1^L, 0, 0, 0)$ with $\mu_1^L \in (0,1)$ Then (1) Period 2 beliefs are $\omega = \left(\frac{1}{2-\mu_1^L}, \frac{1}{2}\right)$. For step (2), we need to make a case distinction:

(2a) If $\tau < \hat{\tau}^M$ or $\left[\frac{1}{2} > \frac{h-l}{1-l} \text{ and } \omega_1/2 > \frac{h-2l}{1-2l}\right]$: The Period 2 proposal is any mix between $(h,0)$ and $(0,h)$. Therefore $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. Then (3a) to support μ there must exist $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)l + \delta qh \\ x_2 &< (1-\delta)l + \delta(1-q)h. \end{aligned}$$

The corresponding area is

$$A(\mu_1^L, 0, 0, 0) = conv\{((1-\delta)\, l, 0), ((1-\delta)\, l, (1-\delta)\, l + \delta h), \\ ((1-\delta)\, l + \delta h, (1-\delta)\, l), ((1-\delta)\, l + \delta h, 0)\}.$$

Note that $A(\mu_1^L, 0, 0, 0) \subset A(1, 0, 0, 0)$.

(b) Otherwise, the Period 2 proposal is (l, l) and so continuation values are uniquely determined as

$$u = \left(l, \frac{l+h}{2}, l, \frac{l+h}{2}\right)$$

Therefore, given μ we have

$$\begin{aligned} x_1 &= l \\ x_2 &< l. \end{aligned}$$

The corresponding area is the line segment

$$A(\mu_1^L, 0, 0, 0) = conv\left\{(l, 0), (l, l)\right\}.$$

Again, it can be shown that, given $\tau > \hat{\tau}^m$, $A(\mu_1^L, 0, 0, 0) \subset A(1, 0, 0, 0)$.

In both cases, $A(\mu_1^L, 0, 0, 0) \subset A(1, 0, 0, 0)$. Thus, each point x_1 that admits continuation equilibria involving $\mu = (\mu_1^L, 0, 0, 0)$ with $\mu_1^L \in (0, 1)$ also admits a continuation equilibrium involving $\mu = (1, 0, 0, 0)$.

Acceptance vector $\mu = (\mu_1^L, 0, 1, 1)$ (1) Period 2 beliefs are $\omega = \left(\frac{1}{2-\mu_1^L}, 1\right)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0, 1]\}$. (3) Given μ there exists $q \in [0, 1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)l + \delta qh \\ x_2 &> (1-\delta q)h. \end{aligned}$$

The corresponding area is

$$\begin{aligned} A(\mu_1^L, 0, 1, 1) = conv\{&((1-\delta)\, l, h), ((1-\delta)\, l, 1-(1-\delta)\, l), \\ &((1-\delta)\, l + \delta h, 1-(1-\delta)\, l - \delta h), ((1-\delta)\, l + \delta h, (1-\delta)\, h)\}. \end{aligned}$$

Acceptance vector $\mu = (\mu_1^L, 0, 1, 0)$ (1) Period 2 beliefs are $\omega = \left(\frac{1}{2-\mu_1^L}, 1\right)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0, 1]\}$. (3) Given μ there exists $q \in [0, 1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)l + \delta qh \\ (1-\delta)l + \delta(1-q)h \le x_2 &\le (1-\delta q)h. \end{aligned}$$

The corresponding area

$$A(\mu_1^L, 0, 1, 0) = conv\{((1-\delta)l, (1-\delta)l + \delta h), ((1-\delta)l, h), ((1-\delta)l + \delta h, (1-\delta)h), ((1-\delta)l + \delta h, (1-\delta)l)\}.$$

Acceptance vector $\mu = (1, \mu_1^H, 0, 0)$ (1) Period 2 beliefs are $\omega = \left(1, \frac{1}{2}\right)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)h + \delta qh \\ x_2 \leq & \ (1-\delta)l + \delta(1-q)h. \end{aligned}$$

The corresponding area is

$$A(1, \mu_1^H, 0, 0) = conv\{(0, (1-\delta)l), (h, (1-\delta)l), ((1-\delta)h, (1-\delta)l + \delta h), ((1-\delta)h, 0)\}.$$

Acceptance vector $\mu = (1, \mu_1^H, 1, 1)$ (1) Period 2 beliefs are $\omega = (1, 1)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)h + \delta qh \\ x_2 \geq & \ (1-\delta q)h. \end{aligned}$$

The corresponding area is

$$A(1, \mu_1^H, 1, 1) = conv\{((1-\delta)h, h), ((1-\delta)h, 1-(1-\delta)h), (h, 1-h), (h, (1-\delta)h)\}.$$

Acceptance vector $\mu = (1, \mu_1^H, 1, 0)$ (1) Period 2 beliefs are $\omega = (1, 1)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. (3) Given μ there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)h + \delta qh \\ (1-\delta)l + \delta(1-q)h \leq x_2 \leq & \ (1-\delta q)h. \end{aligned}$$

The corresponding area is

$$A(1, \mu_1^H, 1, 0) = conv\{((1-\delta)h, (1-\delta)l + \delta h), ((1-\delta)h, h), (h, (1-\delta)h), (h, (1-\delta)l)\}.$$

Voting profiles where both responders are mixing These cases are $\mu = (\mu_1^L, 0, \mu_2^L, 0)$, $\mu = (1, \mu_1^H, \mu_2^L, 0)$, and $\mu = (1, \mu_1^H, 1, \mu_2^H)$.

Acceptance vector $\mu = (\mu_1^L, 0, \mu_2^L, 0)$ (1) Period 2 beliefs are $\omega = \left(\frac{1}{2-\mu_1^L}, \frac{1}{2-\mu_2^L}\right)$. (2) Case distinctions:

(2a) If $\tau < \hat{\tau}^M$ or $\left[\min\{\omega_1, \omega_2\} > \frac{h-l}{1-l} \text{ and } \omega_1\omega_2 > \frac{h-2l}{1-2l}\right]$:
Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$. Then (3a) given μ there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)l + \delta qh \\ x_2 &= (1-\delta)l + \delta(1-q)h. \end{aligned}$$

The corresponding area is

$$A(\mu_1^L, 0, \mu_2^L, 0) = conv\left\{((1-\delta)l, (1-\delta)l + \delta h), ((1-\delta)l + \delta h, (1-\delta)l)\right\}.$$

(2b) Otherwise, the Period 2 proposal is (l, l) and so the continuation values are uniquely determined as

$$u = \left(l, \frac{l+h}{2}, l, \frac{l+h}{2}\right).$$

Then (3b) to support μ, x_1 must satisfy

$$x_1 = x_2 = l.$$

The corresponding area is just a single point:

$$A(\mu_1^L, 0, \mu_2^L, 0) = (l, l).$$

Acceptance vector $\mu = (1, \mu_1^H, \mu_2^L, 0)$ (1) Period 2 beliefs are $\omega = \left(\frac{1}{2-\mu_2^L}, 1\right)$. (2) Again $\mathscr{U}(\mu) = \{(qh, qh, (1-q)h, (1-q)h) : q \in [0,1]\}$, and (3) there exists $q \in [0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)h + \delta qh \\ x_2 &= (1-\delta)l + \delta(1-q)h. \end{aligned}$$

The corresponding area is

$$A(1,\mu_1^H,\mu_2^L,0) = conv\left\{((1-\delta)h,(1-\delta)l+\delta h),(h,(1-\delta)l)\right\}.$$

Acceptance vector $\mu = (1,\mu_1^H,1,\mu_2^H)$ (1) Period 2 beliefs are $\omega=(1,1)$. (2) Again $\mathscr{U}(\mu) = \{(qh,qh,(1-q)h,(1-q)h) : q\in[0,1]\}$, and (3) there exists $q\in[0,1]$ such that

$$\begin{aligned} x_1 &= (1-\delta)h+\delta qh \\ x_2 &= (1-\delta)h+\delta(1-q)h. \end{aligned}$$

The corresponding area is

$$A(1,\mu_1^H,1,\mu_2^H) = conv\left\{((1-\delta)h,h),(h,(1-\delta)h)\right\}.$$

A.3 Proof of Lemma 4

Let $u_P = 1-x_1-x_2$ denote P's utility from immediate passage of the proposal. Denote by $\tilde{p}$ and $\hat{p}$ the probabilities of passage associated with $\tilde{\mu}$ and $\hat{\mu}$, respectively. Similarly, let $\tilde{u}_F$ and $\hat{u}_F$ denote P's expected utilities following proposal failure in the corresponding continuation equilibria. Note that $\tilde{p} < \hat{p} \le 1$ and so the "tilde" equilibrium involves a positive probability of proposal failure. Assume (seeking a contradiction) that P prefers the "tilde" equilibrium. Then we have $\tilde{p}u_P + (1-\tilde{p})\,\tilde{u}_F \ge \hat{p}u_P + (1-\hat{p})\,\hat{u}_F$, or equivalently

$$(1-\tilde{p})\,\tilde{u}_F - (1-\hat{p})\,\hat{u}_F \ge (\hat{p}-\tilde{p})\,u_P > 0.$$

Suppose that $\hat{u}_F \ge \tilde{u}_F$. Then

$$(\hat{p}-\tilde{p})\,\tilde{u}_F = (1-\tilde{p})\,\tilde{u}_F - (1-\hat{p})\,\tilde{u}_F \ge (1-\tilde{p})\,\tilde{u}_F - (1-\hat{p})\,\hat{u}_F \ge (\hat{p}-\tilde{p})\,u_P,$$

thus $\tilde{u}_F \ge u_P$, i.e., P weakly prefers that her proposal fails in the "tilde" equilibrium. In that equilibrium, the Period 2 proposal is *either* **(a)** one of $(h,0)$ and $(0,h)$ *or* **(b)** (l,l) *or* **(c)** a mixture between all three (see Figure 1 and recall that Period 2 beliefs following failure will always satisfy $\omega_i \ge \frac{1}{2}$).

In case (a) the Period 2 proposal allocates h to one responder and passes for sure, thus continuation values satisfy $u_1^H + u_2^H = h$. Since the equilibrium involves a positive probability of failure, both responders must vote N in Period 1 when high. Thus $x_i < (1-\delta)h + \delta u_{iH}$ for $i=1,2$. Therefore,

$x_1 + x_2 < 2(1-\delta)h + \delta(u_1^H + u_2^H) = 2(1-\delta)h + \delta h$. Then $u_P = 1 - x_1 - x_2 > 1 - 2(1-\delta)h - \delta h$. Moreover $\tilde{u}_F = \delta(1-h)$. Thus $\tilde{u}_F \geq u_P$ requires $\delta(1-h) > 1 - 2(1-\delta)h - \delta h$, which is equivalent to $2h > 1$, a contradiction.

Case (b) can occur only if $\tau > \hat{\tau}^m$ and $\tilde{\mu}_{il} < 1$ for $i = 1, 2$ (see Figure 1). Given that P offers (l, l) in Period 2, low type responders would vote Y for sure if $x_i > l$. Thus $x_1 + x_2 \leq 2l$. Hence $u_P = 1 - x_1 - x_2 \geq 1 - 2l > \delta(1-2l) > \tilde{u}_F$, where the last inequality follows from the fact that (l, l) passes with probability less than one in Period 2. Thus $u_P > \tilde{u}_F$, a contradiction.

Case (c) can only occur if P is indifferent between $(h, 0)$, $(0, h)$, and (l, l) in Period 2, implying $2l < h$ and $\tilde{u}_F = \delta\,(1-h)$. Note that each Period 2 proposal will pass if at least one responder is low, and thus $u_1 + u_2 \leq \max\{2l, h\} = h$. P's Period 2 indifference also implies that $\tilde{\mu}_{iL} < 1$, and so $x_i \leq (1-\delta)l + \delta u_i^L$, for $i = 1, 2$. Therefore, $x_1 + x_2 \leq (1-\delta)2l + \delta\,(u_1 + u_2) \leq (1-\delta)2l + \delta h < h$, and so $u_P = 1 - (x_1 + x_2) \geq 1 - h > \delta\,(1-h) = \tilde{u}_F$, a contradiction.

It follows that $\tilde{u}_F > \hat{u}_F$. Then the Period 2 proposal in the "tilde" equilibrium is (l, l) for sure. (To see this, note that $\hat{u}_F \geq \delta(1-h)$ because P can never do worse than offering h to one responder. Thus the only way to achieve $\tilde{u}_F > \hat{u}_F$ is to offer (l, l).) As in case (b), it follows that $\tilde{\mu}_{iL} < 1$ and $x_1 + x_2 \leq 2l$. Hence $u_P = 1 - x_1 - x_2 \geq 1 - 2l$. Further, $\tilde{u}_F = \delta\,(1-2l)\left(1 - \frac{1}{2-\tilde{\mu}_{1l}}\frac{1}{2-\tilde{\mu}_{2l}}\right)$. Since P could offer (l, l) in Period 2 in the "hat" equilibrium, we must have $\hat{u}_F \geq \delta\,(1-2l)\left(1 - \frac{1}{2-\hat{\mu}_{1l}}\frac{1}{2-\hat{\mu}_{2l}}\right)$. Next, it is easy to verify that the probability of Period 1 agreement in the "tilde" equilibrium is $\tilde{p} = 1 - \frac{(2-\tilde{\mu}_{1l})(2-\tilde{\mu}_{2l})}{4}$ and that therefore $(1-\tilde{p})\tilde{u}_F = \left(\frac{3}{4} - \tilde{p}\right)\delta\,(1-2l)$.[16] Similarly $(1-\hat{p})\hat{u}_F \geq$

[16]The probability of Period 1 agreement is the probability that not both responders vote N:

$$\begin{aligned} p &= 1 - \left[\frac{1}{2} + \frac{1-\mu_1^L}{2}\right]\left[\frac{1}{2} + \frac{1-\mu_2^L}{2}\right] \\ &= 1 - \left[\frac{2-\mu_1^L}{2}\right]\left[\frac{2-\mu_2^L}{2}\right] \end{aligned}$$

and thus

$$\begin{aligned} (1-p)\left(1 - \frac{1}{2-\mu_1^L}\frac{1}{2-\mu_2^L}\right) &= \frac{(2-\tilde{\mu}_{1l})(2-\tilde{\mu}_{2l})}{4}\left(1 - \frac{1}{2-\mu_1^L}\frac{1}{2-\mu_2^L}\right) \\ &= \frac{(2-\tilde{\mu}_{1l})(2-\tilde{\mu}_{2l})}{4} - \frac{1}{4} \\ &= 1 - \tilde{p} - \frac{1}{4} = \frac{3}{4} - p \end{aligned}$$

$\left(\frac{3}{4}-\hat{p}\right)\delta\left(1-2l\right)$. Hence

$$\begin{aligned}(1-\tilde{p})\,\tilde{u}_F-(1-\hat{p})\,\hat{u}_F &\leq \left[\left(\frac{3}{4}-\tilde{p}\right)-\left(\frac{3}{4}-\hat{p}\right)\right]\delta\left(1-2l\right)\\ &= (\hat{p}-\tilde{p})\,\delta\left(1-2l\right)\\ &< (\hat{p}-\tilde{p})\,u_P.\end{aligned}$$

Thus $(\hat{p}-\tilde{p})\,u_P > (1-\tilde{p})\,\tilde{u}_F-(1-\hat{p})\,\hat{u}_F$, a contradiction.

B Unanimity rule

B.1 Period 2 proposal (unanimity rule)

Suppose without loss of generality that $\omega_1 \leq \omega_2$. It can be shown that the optimal Period 2 proposal is given by

$$x_2\left(\omega_1,\omega_2\right)=\begin{cases}(l,l) & \text{if } \omega_2<\frac{\tau}{1-2l}\\ (h,h) & \text{if } \omega_1>\frac{\tau}{1-l-h}\\ (l,h) & \text{if } \omega_2>\frac{\tau}{1-2l} \text{ and } \omega_1<\frac{\tau}{1-l-h}\end{cases}.$$

B.2 Period 1 voting (unanimity rule)

We apply the algorithm described in the main text to characterize continuation equilibria following Period 1 proposals allocating strictly less than h to both responders. Recall that high types vote N an all such proposals. Thus, all acceptance probability vectors are of the form $\mu=\left(\mu_1^L,0,\mu_2^L,0\right)$. We therefore focus on *low type* acceptance probability vectors $(\mu_1^L,\mu_2^L)\in[0,1]^2$.

Acceptance vector $\mu^L=(0,0)$ It is easy to see that this is consistent with *any proposal* that allocates strictly less than h to both responders.

Acceptance vector $\mu^L=\left(\mu_1^L,0\right)$ with $\mu_1^L\neq 0$ Then we have (2) $\omega^{NN}=\left(\frac{1}{2-\mu_1^L},\frac{1}{2}\right)$, $\omega^{YN}=\left(0,\frac{1}{2}\right)$ and $\omega^{NY}=(1,0)$, and (3) the relevant continuation values are $u_1^{YN}=l$ and $u_1^{NN}=h$ if $\tau<\hat{\tau}^u$, and $u_1^{NN}=\frac{h+l}{2}$ if $\tau>\hat{\tau}^u$. But (4) $\mu_1^L>0$ requires (given $\mu_2^L=0$) $u_1^{YN}\geq u_1^{NN}$, a contradiction. Thus such acceptance probability vector are *not consistent with any proposal.* (The intuition is that when R2 votes N for sure, the proposal will fail anyway and so R1 cannot possibly accept, since all this does is reveal his low type.)

We can summarize the previous two observations as follows.

Lemma 8. *Certain rejection by both responders constitutes a continuation equilibrium following any Period 1 proposal that allocates strictly less than h to both responders. All other equilibria involve both players voting Y with positive probability as low types.*

The corresponding area of Period 1 proposals x_1 such that there exists a continuation equilibrium involving the acceptance probability vectors $\mu^L = (0,0,0,0)$ thus is

$$A(0,0,0,0) = conv\left\{(0,0),(0,h),(h,h),(h,0)\right\}.$$

Acceptance vector $\mu^L = (1,1)$ This was presented as an example in the main text. We showed there that such continuation equilibria exist if

$$x_i \geq l + 2\delta(h-l) \text{ for } i = 1,2$$

which is consistent with $x_i < h$ only if $\delta < \frac{1}{2}$. If so, these equilibria exist within

$$A(1,0,1,0) = conv\left\{(l+2\delta(h-l), l+2\delta(h-l)), (l+2\delta(h-l), h), (h,h), (h, l+2\delta(h-l))\right\}.$$

Profiles involving mixing To analyse profiles involving mixing, it will be convenient to separately consider the cases $\tau < \hat{\tau}^u$ and $\tau > \hat{\tau}^u$. The first of these is simple because the Period 2 proposal will always allocate h to a responder who voted N in Period 1, and l to a responder who voted Y. As we will see, the area in which mixed equilibria exist is then identical to the area in which the $\mu^L = (1,1)$ equilibrium exists. This will imply that we can disregard these equilibria later in our analysis.

Assume $\tau < \hat{\tau}^u$ and consider $\mu^L = (\mu_1^L, 1)$ with $\mu_1^L \in (0,1)$. Then (1) $\omega^{NN} = \left(\omega_1^N, 1\right)$, $\omega^{YN} = (0,1)$ and $\omega^{NY} = \left(\omega_1^N, 0\right)$ where $\omega_1^N > \bar{\omega}$, so that P offers h or l in Period 2 depending on whether a responder voted N or Y, and that proposal passes. Thus (2) $\left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (h,h,l,h,h,l)$. Then (3) $\mu_1^L \in (0,1)$ requires that

$$x_1 \quad = \quad l + 2\delta(h-l)$$

and $\mu_2^L = 1$ requires that

$$x_2 \quad \geq \quad l + \frac{2\delta}{\mu_1^L}(h-l).$$

For $x_2 \leq h$ we need $\mu_1^L > 2\delta$, requiring $\delta < \frac{1}{2}$. If so, such equilibria exist for each $\mu_1^L \in (2\delta, 1)$. For each such value, the equilibria exist within a line segment

$$A(\mu_1^L, 0, 1, 0) = conv\left\{\left(l + 2\delta(h-l), l + \frac{2\delta}{\mu_1^L}(h-l)\right), (l + 2\delta(h-l), h)\right\}.$$

Given $\mu_1^L \in (2\delta, 1)$, this line segment is a subset of the one connecting $(l + 2\delta(h-l), l + 2\delta(h-l))$ and $(l + 2\delta(h-l), h)$. At each point strictly inside that line segment, there are multiple such equilibria, involving $\mu_1^L \in \left[2\delta\left(\frac{h-l}{x_2-l}\right), 1\right)$. Note that this line segment is exactly equal to the left boundary to the (closed) area in which the $\mu^L = (1,1)$ equilibrium exists. Thus, for each $\mu_1^L \in (2\delta, 1)$, $A(\mu_1^L, 0, 1, 0) \subset A(1, 0, 1, 0)$.

Assume $\tau < \hat{\tau}^u$ and consider $\mu^L = (\mu_1^L, \mu_2^L)$ with both $\mu_i^L \in (0,1)$. Then, (1) $\omega^{NN} = \left(\omega_1^N, \omega_2^N\right)$, $\omega^{YN} = \left(0, \omega_2^N\right)$ and $\omega^{NY} = \left(\omega_1^N, 0\right)$ where both $\omega_i^N > \bar{\omega}$. Any responder voting N (Y) is offered h (l) in Period 2, and that proposal will pass. So (2) $\left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (h, h, l, h, h, l)$ and hence (3) $\mu_i^L \in (0,1)$ requires

$$x_i = l + \frac{2\delta}{\mu_{-iL}}(h-l).$$

This is less than h iff $\mu_{-iL} > 2\delta$, requiring $\delta < \frac{1}{2}$. If so, equilibria involving the responders to vote Y with fixed probabilities $\mu_1^L \in (2\delta, 1)$ and $\mu_2^L \in (2\delta, 1)$ only exist at

$$A(\mu_1^L, 0, \mu_2^L, 0) = \left(l + \frac{2\delta}{\mu_1^L}(h-l), l + \frac{2\delta}{\mu_1^L}(h-l)\right).$$

Consequently, equilibria of this type (when varying the voting Y probabilities over the interval $(2\delta, 1)$) exist within $conv\{(l + 2\delta(h-l), l + 2\delta(h-l)), (l + 2\delta(h-l), h), (h, h), (h, l + 2\delta(h-l))\}$. At each point within this area, $\mu_i^L = 2\delta\left(\frac{h-l}{x_{-i}-l}\right)$. Note that this is decreasing in the share allocated to the other responder, approaching 1 for x_{-i} close to $l + 2\delta(h-l)$ and approaching 2δ as x_{-i} approaches h. Note that this rectangle is identical to $A(1,0,1,0)$ and thus, for all combinations of $\mu_1^L \in (2\delta, 1)$ and $\mu_2^L \in (2\delta, 1)$, $A(\mu_1^L, 0, \mu_2^L, 0) \subset A(1, 0, 1, 0)$.

Assume from now on that $\tau > \hat{\tau}^u$ In this case, it is useful to define a specific acceptance probability, denoted

$$\bar{\mu} = 1 - \frac{1-2h}{h-l} = \frac{3h-l-1}{h-l},$$

with the property that if $\mu_i^L = \bar{\mu}$, then $\omega_{iN} = \bar{\omega} = \frac{h-l}{1-h-l}$. This is relevant because the Period 2 proposals following failure depend on whether $\omega_{iN} \lesseqgtr \bar{\omega}$. (See right panel of Figure 4.)

Acceptance vector $\mu^L = \left(\mu_1^L, 1\right)$ with $\mu_1^L \in (\bar{\mu}, 1)$ Then the analysis is the same as for $\tau < \hat{\tau}^u$, above, except that an additional condition is that μ_1^L must be larger than $\bar{\mu}$. That is, such equilibria exist iff $\delta < \frac{1}{2}$. If so , they exist within

$$A(\mu_1^L, 0, 1, 0) = conv\left\{\left(l + 2\delta(h-l), l + \frac{2\delta}{\mu_1^L}(h-l)\right), (l + 2\delta(h-l), h)\right\}.$$

At each point along this line, there are multiple such equilibria, with $\mu_1^L \in \left[\max\left\{\bar{\mu}, 2\delta\left(\frac{h-l}{x_2-l}\right)\right\}, 1\right)$. Again, for each $\mu_1^L \in (\max\{\bar{\mu}, 2\delta\}, 1)$, $A(\mu_1^L, 0, 1, 0) \subset A(1, 0, 1, 0)$.

Acceptance vector $\mu^L = (\bar{\mu}, 1)$ Then (1) $\omega^{NN} = \left(\omega_1^N, 1\right)$, $\omega^{YN} = (0, 1)$ and $\omega^{NY} = \left(\omega_1^N, 0\right)$, where $\omega_1^N = \frac{1}{2-\mu_1^L} = \bar{\omega}$. Following NN, P is indifferent between (l, h) and (h, h). Suppose in this case P offers (h, h) with probability ρ and (l, h) with $1 - \rho$. Then (2)

$$\left(u_1^{NN}, u_{2L}^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (l + \rho(h-l), h - \frac{1-\rho}{2-\mu_1^L}(h-l), l, h, h, l)$$

Then (3) $\mu_1^L = \bar{\mu} \in (0, 1)$ requires

$$x_1 = l + \delta(1+\rho)(h-l)$$

and $\mu_2^L = 1$ requires

$$x_2 \geq l + \delta\left[\frac{1+\rho}{\bar{\mu}}\right](h-l)$$

This type of equilibrium exists within the line connecting $\left(l + \delta(1+\rho)(h-l), l + \delta\left[\frac{1+\rho}{\bar{\mu}}\right](h-l)\right)$ and $(l + \delta(1+\rho)(h-l), h)$. Such line segment exists if $\delta(1+\rho) < \bar{\mu}$, requiring $\bar{\mu} > \delta$. If so, then every ρ satisfying $\rho < \frac{\bar{\mu}-\delta}{\delta}$ will be associated with such a line segment. At $\rho = 0$, it connects the points $\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right)$ and $(l + \delta(h-l), h)$. At $\rho = \frac{\bar{\mu}-\delta}{\delta}$, it collapses down to the single point $(l + \bar{\mu}(h-l), h)$. As ρ varies between these extremes, we obtain a triangle and thus the equilibrium exists within

$$A(\bar{\mu}, 0, 1, 0) = conv\left\{\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right), (l + \delta(h-l), h), (l + \bar{\mu}(h-l), h)\right\}.$$

At each point within this area, the associated equilibrium involves $\rho = \frac{1}{\delta}\frac{x_1-l}{h-l} - 1$. At the corner $\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right)$, we have $\rho = 0$, and likewise along the line segment to $(l + \delta(h-l), h)$. At the corner $(l + \bar{\mu}(h-l), h)$, we have $\rho = \frac{\bar{\mu}-\delta}{\delta}$.

Acceptance vector $\mu^L = \left(\mu_1^L, 1\right)$ **with** $\mu_{jL} \in (0, \bar{\mu})$ Then (1) $\omega^{NN} = \left(\omega_1^N, 1\right)$, $\omega^{YN} = (0,1)$ and $\omega^{NY} = \left(\omega_1^N, 0\right)$, where $\omega_1^N = \frac{1}{2-\mu_1^L} < \bar{\omega}$. Following NN, P offers (l,h) and so (2) $\left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (l, h - \omega_1^N (h-l), l, h, h, l)$. Thus (3) $\mu_1^L \in (0, \bar{\mu})$ requires that

$$x_1 \quad = \quad l + \delta \left(h - l\right)$$

and $\mu_2^L = 1$ requires that

$$x_2 \geq l + \frac{\delta}{\mu_1^L} \left(h - l\right).$$

For a fixed $\mu_1^L < \bar{\mu}$, equilibria of this type exist within

$$A(\mu_1^L, 0, 1, 0) = conv \left\{ \left(l + \delta(h-l), l + \frac{\delta}{\mu_1^L} \left(h - l\right) \right), \left(l + \delta(h-l), h\right) \right\}.$$

This line segment exists if $\mu_1^L > \delta$ requiring $\bar{\mu} > \delta$. If so, such equilibria exist for all $\mu_1^L \in (\delta, \bar{\mu})$, and along a line segment connecting $\left(l + \delta\left(h-l\right), l + \frac{\delta}{\bar{\mu}}\left(h-l\right)\right)$ and $\left(l + \delta\left(h-l\right), h\right)$. At a given point, μ_1^L can take on any value $\mu_1^L \in \left[\delta \left(\frac{h-l}{x_2 - l}\right), \bar{\mu}\right)$. For each $\mu_1^L \in (\delta, \bar{\mu})$, $A(\mu_1^L, 0, 1, 0) \subset A(\bar{\mu}, 0, 1, 0)$.

Acceptance vector $\mu^L = (\mu_1^L, \mu_2^L)$ **with both** $\mu_i^L \in (\bar{\mu}, 1)$ Then (1) $\omega_i^N > \bar{\omega}$ for both i, and so as in the case $\tau < \hat{\tau}^u$, $\left(u_1^{NN}, u_2^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (h, h, l, h, h, l)$ and (3) $x_i = l + \frac{2\delta}{\mu_{-iL}}\left(h - l\right)$, requiring $\mu_i > 2\delta$. So, given $\mu_1^L \in (\max\{\bar{\mu}, 2\delta\}, 1)$ and $\mu_2^L \in (\max\{\bar{\mu}, 2\delta\}, 1)$, the equilibrium exists at

$$A(\mu_1^L, 0, \mu_2^L, 0) = \left(l + \frac{2\delta}{\mu_2^L}\left(h - l\right), l + \frac{2\delta}{\mu_1^L}\left(h - l\right) \right).$$

By varying the voting Y probabilities, we obtain the area in which such types of equilibria exist: $conv \left\{ \left(l + 2\delta\left(h-l\right), l + 2\delta\left(h-l\right)\right), \left(l + 2\delta(h-l), h\right), \left(h, h\right), \left(h, l + 2\delta(h-l)\right) \right\}$ if $\bar{\mu} < 2\delta$ and

$$\begin{aligned} conv\{ &\left(l + 2\delta\left(h-l\right), l + 2\delta\left(h-l\right)\right), \left(l + 2\delta(h-l), h\right), \\ &\qquad \left(l + \frac{2\delta}{\bar{\mu}}\left(h-l\right), l + \frac{2\delta}{\bar{\mu}}\left(h-l\right)\right), \left(h, l + 2\delta(h-l)\right) \} \end{aligned}$$

if $\bar{\mu} > 2\delta$. At each point within this area, $\mu_i^L = 2\delta\left(\frac{h-l}{x_{-i} - l}\right)$. Again, for each combination of $\mu_1^L \in (\max\{\bar{\mu}, 2\delta\}, 1)$ and $\mu_2^L \in (\max\{\bar{\mu}, 2\delta\}, 1)$, $A(\mu_1^L, 0, \mu_2^L, 0) \subset A(1, 0, 1, 0)$.

Acceptance vector $\mu^L = (\mu_1^L, \mu_2^L)$ **with** $0 < \mu_1^L < \mu_2^L < \bar{\mu}$ Then (1) $\frac{1}{2} < \omega_1^N < \omega_2^N < \bar{\omega}$. P offers (l,h) following NN. Thus (2)

$$\left(u_1^{NN}, u_{2L}^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (l, h - \frac{h-l}{2 - \mu_1^L}, l, h, h, l).$$

Then (3) mixing by both responders requires

$$x_1 = l + \delta (h - l)$$

and

$$x_2 = l + \frac{\delta}{\mu_1^L} (h - l).$$

Then $x_2 < h$ requires $\mu_1^L > \delta$. Thus, given $\mu_1^L \in (\delta, \bar{\mu})$, the equilibrium exists at

$$A(\mu_1^L, 0, \mu_2^L, 0) = \left(l + \delta (h - l), l + \frac{\delta}{\mu_1^L} (h - l) \right).$$

By varying the voting Y probabilities, we obtain the area in which such types of equilibria exist: *conv* $\left\{ \left(l + \delta (h - l), l + \frac{\delta}{\bar{\mu}} (h - l) \right), (l + \delta (h - l), h) \right\}$. (These equilibria exist *strictly* between those points.) At each point, $\mu_1^L = \delta \left(\frac{h-l}{x_2 - l} \right)$ and $\mu_2^L \in (\mu_1^L, \bar{\mu})$. For each combination of $\mu_1^L \in (\delta, \bar{\mu})$ and $\mu_2^L \in (\mu_1^L, \bar{\mu})$, $A(\mu_1^L, 0, \mu_2^L, 0) \subset A(\bar{\mu}, 0, 1, 0)$.

Acceptance vector $\mu^L = (\mu_1^L, \mu_2^L)$ with $0 < \mu_1^L = \mu_2^L = \mu < \bar{\mu}$ Then (1) $\omega_1^N = \omega_2^N \in (\frac{1}{2}, \bar{\omega})$. P is indifferent between (l, h) and (h, l) following NN. Assume he offers (h, l) with prob ρ. Then (2)

$$\left(u_1^{NN}, u_{2L}^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN} \right) = (l + \rho \frac{1 - \mu}{2 - \mu} (h - l), l + (1 - \rho) \frac{1 - \mu}{2 - \mu} (h - l), l, h, h, l).$$

Then (3) mixing by both responders requires

$$\begin{aligned} x_1 &= l + \left(1 + \rho \frac{1 - \mu}{\mu} \right) \delta (h - l) \\ x_2 &= l + \left(1 + (1 - \rho) \frac{1 - \mu}{\mu} \right) \delta (h - l). \end{aligned}$$

For $x_i < h$, we need $\frac{\delta \rho}{1 - \delta(1 - \rho)} < \mu$ and $\frac{\delta(1 - \rho)}{1 - \delta \rho} < \mu$. I.e., a necessary condition for equilibria of this type to exist is that there exists $\rho \in [0, 1]$ such that $\mu > \max \left\{ \frac{\delta \rho}{1 - \delta(1 - \rho)}, \frac{\delta(1 - \rho)}{1 - \delta \rho} \right\}$. The first lower bound is increasing and the second decreasing in ρ, and they are completely symmetric. For a given μ, the inequality can therefore hold at *some* ρ iff it holds for $\rho = \frac{1}{2}$, giving the condition

$$\mu > \frac{\delta}{2 - \delta},$$

which in turn is compatible with $\mu < \bar{\mu}$ iff

$$\frac{\delta}{2 - \delta} < \frac{3h - l - 1}{h - l}.$$

Under the last condition, there exist μ and ρ such that these type of equilibria exist. If so, the possible range for μ is $\left(\frac{\delta}{2-\delta},\bar{\mu}\right)$. For each μ in this range, there is a range for ρ such that both $x_i < h$. This is $\rho \in [0,1]$ if $\delta < \mu$ and $\rho \in \left(\frac{\delta-\mu}{\delta(1-\mu)},\frac{\mu}{1-\mu}\frac{1-\delta}{\delta}\right)$ if $\delta \geq \mu$, which is nonempty given $\mu > \frac{\delta}{2-\delta}$. At $\mu = \frac{\delta}{2-\delta}$, that line segment collapses to the single point (h,h). Thus, given $\mu \in (\frac{\delta}{2-\delta},\delta)$, the equilibrium exists within (at $\mu = \frac{\delta}{2-\delta}$, the line segment collapses to the single point (h,h))

$$A(\mu,0,\mu,0) = conv\left\{\left(l+\left(1+\frac{\delta-\mu}{\mu\delta}\right)\delta\left(h-l\right),h\right),\left(h,l+\left(1+\frac{\delta-\mu}{\mu\delta}\right)\left(h-l\right)\right)\right\}$$

and given $\mu \in (\delta,\bar{\mu})$, within

$$A(\mu,0,\mu,0) = \left\{\left(l+\delta\left(h-l\right),l+\frac{\delta}{\mu}\left(h-l\right)\right),\left(l+\frac{\delta}{\mu}\left(h-l\right),l+\delta\left(h-l\right)\right)\right\}.$$

If $\bar{\mu} < \delta$, as μ increases the line segments are all diagonals sloping down within the square to the southwest of (h,h). At $\mu = \bar{\mu}$, the last such line connects $\left(l+\left(1+\frac{\delta-\bar{\mu}}{\bar{\mu}\delta}\right)\delta\left(h-l\right),h\right)$ and $\left(h,l+\left(1+\frac{\delta-\bar{\mu}}{\bar{\mu}\delta}\right)\delta\left(h-l\right)\right)$. In that case we obtain a triangle with corners given by those two points and (h,h). If $\bar{\mu} > \delta$, then at $\mu = \delta$ the line segment connects the points $(l+\delta\left(h-l\right),h)$ and $(h,l+\delta\left(h-l\right))$, and as μ increases further it eventually connects $\left(l+\delta\left(h-l\right),l+\frac{\delta}{\bar{\mu}}\left(h-l\right)\right)$ and $\left(l+\frac{\delta}{\bar{\mu}}\left(h-l\right),l+\delta\left(h-l\right)\right)$. So in this case we obtain a pentagon which is the convex hull of those four points and (h,h). (A simpler way to describe this may be that it is the area where both $x_i \in (l+\delta(h-l),h)$ and $x_1+x_2 > l+h+\delta(h-l)$.) Equilibria of this type exist in the interior of the area. At each such point, μ is determined by the sum of offers x_1+x_2:

$$\mu = \frac{\delta\left(h-l\right)}{x_1+x_2-2l-\delta\left(h-l\right)}.$$

(Note that, paradoxically, the acceptance probability is *decreasing* in the sum of offers.) ρ is likewise determined but irrelevant for our subsequent analysis because P's EU is independent of ρ.

Acceptance vector $\mu^L = (\bar{\mu},\mu_2^L)$ with $\mu_2^L \in (\bar{\mu},1)$ Then (1) $\omega^{NN} = (\bar{\omega},\omega_2^N)$ with $\omega_2^N > \bar{\omega}$. Following NN, P is indifferent between (l,h) and (h,h). Suppose she proposes (h,h) with probability ρ. Then (2)

$$\left(u_1^{NN},u_{2L}^{NN},u_1^{YN},u_2^{NY},u_1^{NY},u_2^{YN}\right) = (l+\rho\left(h-l\right),h-\frac{1-\rho}{2-\bar{\mu}}\left(h-l\right),l,h,h,l).$$

Therefore, (3) mixing by both responders requires

$$\begin{aligned} x_1 &= l + \left[1 + \frac{2-\mu_2^L}{\mu_2^L}\rho\right]\delta(h-l) \\ x_2 &= l + \frac{1+\rho}{\bar{\mu}}\delta(h-l). \end{aligned}$$

It is easy to verify that $x_2 > x_1$. Then $x_i < h$ requires $\rho < \frac{\bar{\mu}-\delta}{\delta}$, giving us the necessary condition $\delta < \bar{\mu}$ and the range $\rho \in \left(0, \min\left\{\frac{\bar{\mu}-\delta}{\delta}, 1\right\}\right)$. So, given $\mu_2^L \in (\bar{\mu}, 1)$ and $\delta < \bar{\mu}/2$, the equilibrium exists within

$$A(\mu, 0, \mu, 0) = conv\left\{\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right), \left(l + \frac{2\delta}{\mu_2^L}(h-l), l + \frac{2\delta}{\bar{\mu}}(h-l)\right)\right\},$$

and, given $\mu_2^L \in (\bar{\mu}, 1)$ and $\delta \geq \bar{\mu}/2$, within

$$A(\mu, 0, \mu, 0) = conv\left\{\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right), \left(l + \frac{2\delta}{\mu_2^L}(h-l), l + \frac{2\delta}{\bar{\mu}}(h-l)\right)\right\}.$$

If $\delta < \bar{\mu}/2$, varying $\mu_2^L \in (\bar{\mu}, 1)$ gives us a triangle with corners $\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right)$, $\left(l + \frac{2\delta}{\bar{\mu}}(h-l), l + \frac{2\delta}{\bar{\mu}}(h-l)\right)$, and $\left(l + 2\delta(h-l), l + \frac{2\delta}{\bar{\mu}}(h-l)\right)$. If $\delta \in (\bar{\mu}/2, \bar{\mu})$, varying $\mu_2^L \in (\bar{\mu}, 1)$ gives us a triangle with corners $\left(l + \delta(h-l), l + \frac{\delta}{\bar{\mu}}(h-l)\right)$, $\left(l + \left[1 + \frac{2-\bar{\mu}}{\bar{\mu}}\frac{\bar{\mu}-\delta}{\delta}\right]\delta(h-l), h\right)$, and $(l + \bar{\mu}\delta(h-l), h)$. At each point within this area, the associated probabilities are determined by the equations for x_i above:

$$\begin{aligned} \mu_2^L &= 2\frac{\bar{\mu}(x_2 - l) - \delta(h-l)}{\bar{\mu}(x_2 - l) + (x_1 - l)} \\ \rho &= \frac{\bar{\mu}}{\delta}\frac{x_2 - l}{h - l} - 1. \end{aligned}$$

Acceptance vector $\mu^L = (\bar{\mu}, \bar{\mu})$ Then (1) $\omega^{NN} = (\bar{\omega}, \bar{\omega})$ and P is indifferent between (l,h), (h,l), and (h,h) following NN. Suppose in this case he offers (h,l) with prob ρ, (l,h) with γ and (h,h) with $1-\rho-\gamma$. Then (2)

$$\left(u_1^{NN}, u_{2L}^{NN}, u_1^{YN}, u_2^{NY}, u_1^{NY}, u_2^{YN}\right) = (h - \left(\gamma + \frac{\rho}{2-\bar{\mu}}\right)(h-l), h - \left(\rho + \frac{\gamma}{2-\bar{\mu}}\right), l, h, h, l).$$

Then (3) mixing by both responders requires

$$x_1 = l + (2 - \rho - \gamma - (1-\bar{\mu})\gamma)\frac{\delta}{\bar{\mu}}(h-l)$$

$$x_2 = l + (2 - \rho - \gamma - (1 - \bar{\mu})\rho) \frac{\delta}{\bar{\mu}} (h - l) .$$

Then $x_i < h$ requires

$$\frac{\bar{\mu}}{\delta} > 2 - \rho - \gamma - (1 - \bar{\mu}) \min \{\gamma, \rho\} .$$

For a given sum $\rho + \gamma$, the best chance to satisfy this is $\rho = \gamma$, in which case it can be satisfied iff there exists $\rho \in (0, \frac{1}{2})$ such that $\frac{\bar{\mu}}{\delta} > 2 - (3 - \bar{\mu})\rho$. Here, the best case is $\rho = \frac{1}{2}$. Thus, a necessary and sufficient condition for the existence of such equilibria is $\frac{\bar{\mu}}{\delta} > \frac{1}{2} + \bar{\mu}/2$, or

$$\delta < \frac{2\bar{\mu}}{\bar{\mu} + 1}.$$

If so, the region in which these equilibria exist is non-empty. The region arises by varying ρ and γ. If $\delta < \bar{\mu}/2$, the northeast corner of the triangle that characterizes the equilibrium points arises from the case where P always offers (h, h) in Period 2, i.e., $(\rho, \gamma) = (0, 0)$, leading to the point $\left(l + \frac{2\delta}{\bar{\mu}} (h - l) , l + \frac{2\delta}{\bar{\mu}} (h - l)\right)$. The triangle is completed by the corners where $(\rho, \gamma) = (1, 0)$ and $(\rho, \gamma) = (0, 1)$ and thus the equilibrium, given $\delta < \bar{\mu}/2$, exists within

$$A(\bar{\mu}, 0, \bar{\mu}, 0) = conv\{\left(l + \delta (h - l) , l + \frac{\delta}{\bar{\mu}} (h - l)\right), \left(l + \frac{2\delta}{\bar{\mu}} (h - l) , l + \frac{2\delta}{\bar{\mu}} (h - l)\right), \left(l + \frac{\delta}{\bar{\mu}} (h - l) , l + \delta (h - l)\right)\}.$$

If $\delta \in (\bar{\mu}/2, \bar{\mu})$, then $(\rho, \gamma) = (0, 0)$ is not feasible (it would require $x_i > h$). Consequently, the northeast corner now is (h, h). The upper corner to the west of (h, h) is where $x_2 = h$ and $\rho = 0$, leading to $\left(h - \frac{(2\delta - \bar{\mu})(1 - \bar{\mu})}{\bar{\mu}} (h - l) , h\right)$. By symmetry, the corner to the south of (h, h) is where $x_1 = h$ and $\gamma = 0$, leading to $\left(h, h - \frac{(2\delta - \bar{\mu})(1 - \bar{\mu})}{\bar{\mu}} (h - l)\right)$. The other two corners in the southwest stay as before and thus the equilibrium, given $\delta \in (\bar{\mu}/2, \bar{\mu})$, exists within

$$A(\bar{\mu}, 0, \bar{\mu}, 0) = conv\{\left(l + \delta (h - l) , l + \frac{\delta}{\bar{\mu}} (h - l)\right), \left(h - \frac{(2\delta - \bar{\mu}) (1 - \bar{\mu})}{\bar{\mu}} (h - l) , h\right), (h, h) , \left(l + \frac{2\delta}{\bar{\mu}} (h - l) , l + \frac{2\delta}{\bar{\mu}} (h - l)\right), \left(h, h - \frac{(2\delta - \bar{\mu}) (1 - \bar{\mu})}{\bar{\mu}} (h - l)\right), \left(l + \frac{\delta}{\bar{\mu}} (h - l) , l + \delta (h - l)\right)\}.$$

If $\delta \in \left(\bar{\mu}, \frac{2\bar{\mu}}{\bar{\mu}+1}\right)$, the northeast corner again is (h, h). Now, $(\rho, \gamma) = (1, 0)$ and $(\rho, \gamma) = (0, 1)$ are not feasible as they would require $x_2 > h$ and $x_1 > h$, respectively. The upper corner to the west of (h, h) is where $x_2 = h$ and $\rho + \gamma = 1$ (note that $x_1 + x_2 = 2l + \delta \frac{1+\bar{\mu}}{\bar{\mu}} (h - l)$ if $\rho + \gamma = 1$), leading to $\left(l + \left(\delta \frac{1+\bar{\mu}}{\bar{\mu}} - 1\right) (h - l) , h\right)$. By symmetry, the corner to the south of (h, h) is where $x_1 = h$ and

$\rho+\gamma=1$, leading to $\left(h, l+\left(\delta\frac{1+\bar{\mu}}{\bar{\mu}}-1\right)(h-l)\right)$. Thus the equilibrium, given $\delta\in\left(\bar{\mu}, \frac{2\bar{\mu}}{\bar{\mu}+1}\right)$, exists within

$$A(\bar{\mu},0,\bar{\mu},0)=conv\left\{\left(l+\left(\delta\frac{1+\bar{\mu}}{\bar{\mu}}-1\right)(h-l), h\right), (h,h), \left(h, l+\left(\delta\frac{1+\bar{\mu}}{\bar{\mu}}-1\right)(h-l)\right)\right\}.$$

To sum up, each of these areas are part of the triangle that can be described by the convex hull $conv\left\{\left(l+\delta(h-l), l+\frac{\delta}{\bar{\mu}}(h-l)\right), \left(l+\frac{2\delta}{\bar{\mu}}(h-l), l+\frac{2\delta}{\bar{\mu}}(h-l)\right), \left(l+\frac{\delta}{\bar{\mu}}(h-l), l+\delta(h-l)\right)\right\}$. There, the conditions $x_1<h$ and $x_2<h$ determine the size of the triangle where an equilibrium with an acceptance vector of $\mu=(\bar{\mu},0,\bar{\mu},0)$ exists. At each point within these areas, the associated probabilities are $\rho=\frac{2(1-\bar{\mu})\delta(h-l)+\bar{\mu}(x_1-2x_2+l+\bar{\mu}(x_2-l))}{(3-\bar{\mu})(1-\bar{\mu})\delta(h-l)}$ and $\gamma=\frac{2(1-\bar{\mu})\delta(h-l)+\bar{\mu}(x_2-2x_1+l+\bar{\mu}(x_1-l))}{(3-\bar{\mu})(1-\bar{\mu})\delta(h-l)}$ (determined by the equations for x_i above).

B.3 Selection of voting equilibria (unanimity rule)

P generally prefers the equilibrium with the larger acceptance probability. If she does not, then she will not make such proposals in Period 1. This we will show below. We exclude equilibrium areas that are fully included in another equilibrium area that is associated with a larger probability of passage and where (at least) at the "cheapest" points of the equilibrium area associated with the lower probability of passage, P indeed prefers the equilibrium with the larger probability of passage:

- $A\left(\mu_1^L,0,1,0\right)\subset A(\bar{\mu},0,1,0)$ for all $\mu_1^L\in(\delta,\bar{\mu})$ and for each of the cheapest points of $A(\mu_1^L,0,1,0)$ where $\mu_1^L\in(\delta,\bar{\mu})$, P prefers the continuation equilibrium associated with $\mu=(\bar{\mu},0,1,0)$: Suppose $\tilde{x}^1\in A(\tilde{\mu},0,1,0)=conv\left\{\left(l+\delta(h-l), l+\frac{\delta}{\tilde{\mu}}(h-l)\right), (l+\delta(h-l), h)\right\}$. Then at the cheapest of such equilibria, we have $EU_P\left(\tilde{x}^1,\tilde{\mu}\right)=\frac{\tilde{\mu}}{4}\left(1-2l-\frac{1+\tilde{\mu}}{\tilde{\mu}}\delta(h-l)\right)+\frac{\delta}{2}(1-h-l)+\frac{1}{4}(1-\tilde{\mu})\delta(1-h-l)$ and $\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}}=\frac{1}{4}(1-2l-\delta(h-l))-\frac{1}{4}\delta(1-h-l)>0$, which can easily be shown to be true.

- $A(\mu_1^L,0,\mu_2^L,0)\subset A(\bar{\mu},0,1,0)$ for all combinations of $\mu_1^L\in(\delta,\bar{\mu})$ and $\mu_2^L\in(\mu_1^L,\bar{\mu})$. For $x_1\in A(\mu_1^L,0,\mu_2^L,0)=\left(l+\delta(h-l), l+\frac{\delta}{\mu_1^L}(h-l)\right)$, P prefers the continuation equilibrium associated with $\mu=(\bar{\mu},0,1,0)$: $EU_P\left(x_1,\mu=\left(\mu_1^L,0,\tilde{\mu},0\right)\right)=\frac{\mu_1^L\tilde{\mu}}{4}\left(1-2l-\frac{1+\mu_1^L}{\mu_1^L\tilde{\mu}}\delta(h-l)\right)+\frac{\delta}{2}\left(\tilde{\mu}+\mu_1^L-\mu_1^L\tilde{\mu}\right)(1-h-l)+\left(2-\mu_1^L\right)(1-\tilde{\mu})\frac{\delta}{4}(1-h-l)$ and $\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}}=\frac{\mu_1^L}{4}(1-2l)+\frac{\delta}{2}\left(1-\mu_1^L\right)(1-h-l)-\left(2-\mu_1^L\right)\frac{\delta}{4}(1-h-l)>0$ which can be shown to be true. Thus P prefers $\mu=\left(\mu_1^L,0,1,0\right)$ over $\mu=\left(\mu_1^L,0,\tilde{\mu},0\right)$. In addition, we know from above that P prefers $\mu=(\bar{\mu},0,1,0)$ over $\mu=\left(\mu_1^L,0,1,0\right)$ when $\mu_1^L\in(\delta,\bar{\mu})$.

- $A(\mu_1^L,0,1,0)\subset A(1,0,1,0)$ for all $\mu_1^L\in(\max\{\bar{\mu},2\delta\},1)$. For each of the cheapest points of

$A(\mu_1^L,0,1,0)$ where $\mu_1^L \in (\max\{\bar{\mu},2\delta\},1)$, P prefers the continuation equilibrium associated with $\mu=(1,0,1,0)$: Suppose that $\tilde{x}^1 \in A(\tilde{\mu},0,1,0) = conv\left\{\left(l+2\delta(h-l), l+\frac{2\delta}{\tilde{\mu}}(h-l)\right),(l+2\delta(h-l),h)\right\}$. So at the cheapest of such equilibria, we have $EU_P\left(\tilde{x}^1,\tilde{\mu}\right) = \frac{\tilde{\mu}}{4}\left(1-2l-2\frac{1+\tilde{\mu}}{\tilde{\mu}}\delta(h-l)\right)+\frac{\delta}{2}(1-h-l)+\frac{1}{2}\left(1-\frac{\tilde{\mu}}{2}\right)\delta(1-2h)$ and $\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}} = \frac{1}{4}(1-2l-2\delta(h-l))-\frac{1}{4}\delta(1-2h) > 0$ which can easily be shown to be true.

- $A(\mu_1^L,0,\mu_2^L,0) \subset A(1,0,1,0)$ for all combinations of $\mu_1^L \in (\max\{\bar{\mu},2\delta\},1)$ and $\mu_2^L \in (\max\{\bar{\mu},2\delta\},1)$. For $x_1 \in A(\mu_1^L,0,\mu_2^L,0) = \left(l+\frac{2\delta}{\mu_2^L}(h-l), l+\frac{2\delta}{\mu_1^L}(h-l)\right)$, P prefers the continuation equilibrium associated with $\mu=(1,0,1,0)$: We have $EU_P\left(x_1,\mu=\left(\mu_1^L,0,\tilde{\mu},0\right)\right) = \frac{\mu_1^L\tilde{\mu}}{4}\left(1-2l-2\frac{\mu_1^L+\tilde{\mu}}{\mu_1^L\tilde{\mu}}\delta(h-l)\right)+\frac{\delta}{2}\left(\tilde{\mu}+\mu_1^L-\mu_1^L\tilde{\mu}\right)(1-h-l)+\left(1-\frac{\mu_1^L}{2}\right)\left(1-\frac{\tilde{\mu}}{2}\right)\delta(1-2h)$ and thus $\frac{\partial EU_P\left(x_1,\mu=\left(\mu_1^L,0,\tilde{\mu},0\right)\right)}{\partial\tilde{\mu}} = \frac{\mu_1^L}{4}(1-2l)-\frac{\delta}{2}(h-l)+\frac{\delta}{2}\left(1-\mu_1^L\right)(1-h-l)-\left(2-\mu_1^L\right)\frac{\delta}{4}(1-2h) > 0$ which can be shown to be true. Thus P prefers $\mu=\left(\mu_1^L,0,1,0\right)$ over $\mu=\left(\mu_1^L,0,\tilde{\mu},0\right)$. In addition, we know from above that P prefers $\mu=(1,0,1,0)$ over $\mu=\left(\mu_1^L,0,1,0\right)$.

- $A(\mu_1^L,0,1,1) \subset A(1,0,1,1)$ for all $\mu_1^L \in (0,1)$.
For $x_1 \in A(\mu_1^L,0,1,1) = \begin{cases}(l+\delta(h-l),h) & \text{if } \tau<\hat{\tau}^u \\ \left(l+\frac{\delta}{2}(h-l),h\right) & \text{if } \tau>\hat{\tau}^u\end{cases}$, P prefers the continuation equilibrium associated with $\mu=(1,0,1,1)$:
(a) Suppose $\tau<\hat{\tau}^u$: $EU_P\left(\tilde{x}^1,\tilde{\mu}\right) = \frac{\tilde{\mu}}{2}(1-l-\delta(h-l)-h)+\left(1-\frac{\tilde{\mu}}{2}\right)\delta(1-2h)$ and $\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}} = \frac{1}{2}(1-l-\delta(h-l)-h)-\frac{\delta}{2}(1-2h) > 0$ which is true because $1-l-h > \delta(1-h-l)$.
(b) Suppose $\tau>\hat{\tau}^u$: $EU_P\left(\tilde{x}^1,\tilde{\mu}\right) = \frac{\tilde{\mu}}{2}\left(1-l-\frac{\delta}{2}(h-l)-h\right)+\left(1-\frac{\tilde{\mu}}{2}\right)\frac{\delta}{2}(1-h-l)$ and $\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}} = \frac{1}{2}\left(1-l-\frac{\delta}{2}(h-l)-h\right)-\frac{\delta}{4}(1-h-l) > 0$ which can be shown to be true. Thus, for $x_1 = A(\mu_1^L,0,1,1)$, P prefers the continuation equilibrium associated with $\mu=(1,0,1,1)$.

In addition, one can show that

- $A(\mu,0,\mu,0) \subset$
$(A(\bar{\mu},0,\bar{\mu},0) \cup A(\bar{\mu},0,\hat{\mu},0) \cup A(\hat{\mu},0,\bar{\mu},0) \cup A(\bar{\mu},0,1,0) \cup A(1,0,\bar{\mu},0) \cup A(1,0,1,0))$ for all $\mu \in \left(\frac{\delta}{2-\delta},\bar{\mu}\right)$ with $\hat{\mu} = \frac{2\bar{\mu}(x_1-l)-\delta\tau}{x_2-l+\bar{\mu}(x_1-l)-\delta\tau}$. For each of the cheapest points of $A(\mu,0,\mu,0)$, i.e., where $\tilde{x}_1+\tilde{x}_2 = l+h+\delta(h-l)$, P prefers the continuation equilibrium associated with $\mu = (\bar{\mu},0,\bar{\mu},0)$: We have $EU_P\left(\tilde{x}^1,\mu=(\tilde{\mu},0,\tilde{\mu},0)\right) = \frac{\tilde{\mu}^2}{4}(1-h-l-\delta(h-l))+2\delta\frac{\tilde{\mu}}{2}\left(1-\frac{\tilde{\mu}}{2}\right)(1-h-l)+\left(1-\frac{\tilde{\mu}}{2}\right)\left(1-\frac{\tilde{\mu}}{2}\right)\delta\frac{1-\tilde{\mu}}{2-\tilde{\mu}}(1-h-l)$ and thus

$\frac{\partial EU_P(x_1,\tilde{\mu})}{\partial\tilde{\mu}} = \frac{\tilde{\mu}}{2}\left(1-h-l-\delta\left(h-l\right)\right)+\left(1-2\tilde{\mu}\right)\frac{\delta}{4}\left(1-h-l\right) > 0$ which can be shown to hold true.

Consequently, we can solve multiplicity of equilibrium points (i.e., at a given point in Period 1 proposal space there are multiple voting Y probabilities leading to an equilibrium) by laying focus on the equilibrium that is associated with the largest probability of Period 1 passage. So the areas specified in Proposition 3 (and Figure 4) are the areas, together with the respective voting Y probabilities, that are not part of any other area associated with a larger voting Y probability (w.l.o.g. suppose that $\mu_1^L \leq \mu_2^L$ and define $\bar{\mu} = \frac{3h-l-1}{h-l} = 1-\frac{1-2h}{h-l}$):

1. Region 1: Both responders vote N irrespective of their type, i.e. $\mu = (0,0,0,0)$. The area that is not part of any of the other equilibrium areas.

2. Region 2: $A\left(0,0,1,1\right) = \begin{cases} conv\left\{\left(0,h\right),\left(l+\frac{\delta}{2}\tau,h\right)\right\} & \tau > \hat{\tau}^u \\ conv\left\{\left(0,h\right),\left(h,h\right)\right\} & \tau < \hat{\tau}^u \end{cases}$ (see Lemma 7) joint with its symmetric counterpart.

3. Region 3: $A\left(\bar{\mu},0,\bar{\mu},0\right) = conv\left\{\left(l+\delta\tau,l+\frac{\delta}{\bar{\mu}}\tau\right),\left(l+2\frac{\delta}{\bar{\mu}}\tau,l+2\frac{\delta}{\bar{\mu}}\tau\right),\left(l+\frac{\delta}{\bar{\mu}}\tau,l+\delta\tau\right)\right\}$ if $\delta \in \left(0,\frac{\bar{\mu}}{2}\right)$,

$$A\left(\bar{\mu},0,\bar{\mu},0\right) = conv\{\left(l+\delta\tau,l+\frac{\delta}{\bar{\mu}}\tau\right),\left(h-\frac{\left(2\delta-\bar{\mu}\right)\left(1-\bar{\mu}\right)}{\bar{\mu}}\tau,h\right),\left(h,h\right), \left(h,h-\frac{\left(2\delta-\bar{\mu}\right)\left(1-\bar{\mu}\right)}{\bar{\mu}}\tau\right),\left(l+\frac{\delta}{\bar{\mu}}\tau,l+\delta\tau\right)\}$$

if $\delta \in \left(\frac{\bar{\mu}}{2},\bar{\mu}\right)$, and $A\left(\bar{\mu},0,\bar{\mu},0\right) = conv\left\{\left(l+\left(\delta\frac{1+\bar{\mu}}{\bar{\mu}}-1\right)\tau,h\right),\left(h,h\right),\left(h,l+\left(\delta\frac{1+\bar{\mu}}{\bar{\mu}}-1\right)\tau\right)\right\}$ if $\delta \in \left(\bar{\mu},\frac{2\bar{\mu}}{1+\bar{\mu}}\right)$ (allocating in total more than $2l+\frac{1+\bar{\mu}}{\bar{\mu}}\delta\tau$).

4. Region 4:

$$A\left(\bar{\mu},0,\hat{\mu},0\right) = \begin{pmatrix} conv\left\{\left(l+\delta\tau,l+\frac{\delta}{\bar{\mu}}\tau\right),\left(l+\frac{2}{\bar{\mu}}\delta\tau,l+\frac{2}{\bar{\mu}}\delta\tau\right),\left(l+2\delta\tau,l+\frac{2}{\bar{\mu}}\delta\tau\right)\right\} & \text{if } \delta \in \left(0,\frac{\bar{\mu}}{2}\right) \\ conv\left\{\left(l+\delta\tau,l+\frac{\delta}{\bar{\mu}}\tau\right),\left(l+\left(\frac{\left(\bar{\mu}-\delta\right)\left(2-\bar{\mu}\right)}{\bar{\mu}}+\delta\right)\tau,h\right),\left(l+\bar{\mu}\tau,h\right)\right\} & \text{if } \delta \in \left(\frac{\bar{\mu}}{2},\bar{\mu}\right) \end{pmatrix}$$

(where $\hat{\mu} = \frac{2\bar{\mu}(x_1-l)-\delta\tau}{x_2-l+\bar{\mu}(x_1-l)-\delta\tau}$) joint with its symmetric counterpart.

5. Region 5:

$$A(\bar{\mu},0,1,0) = conv\{\left(l+\delta\tau, l+\frac{\delta}{\bar{\mu}}\tau\right), (l+\delta\tau, h), (\min\{l+2\delta\tau, l+\bar{\mu}\tau\}, h), \left(\min\{l+2\delta\tau, l+\bar{\mu}\tau\}, \min\left\{l+2\frac{\delta}{\bar{\mu}}\tau, h\right\}\right)\}$$

(given $\delta < \bar{\mu}$) joint with its symmetric counterpart.

6. Region 6: $A(1,0,1,0) = conv\{(l+2\delta\tau, l+2\delta\tau), (l+2\delta\tau, h), (h,h), (h, l+2\delta\tau)\}$ (given $\delta < \frac{1}{2}$).

7. Region 7: $A(1,0,1,1) = \begin{cases} conv\left\{\left(l+\frac{\delta}{2}(h-l), h\right), (h,h)\right\} & \tau > \hat{\tau}^u \\ conv\{(l+\delta(h-l), h), (h,h)\} & \tau < \hat{\tau}^u \end{cases}$ joint with its symmetric counterpart.

8. Region 8: $\mu = (1,1,1,1)$ and allocating at least h to each responder.