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Negative Emotion Accumulation and Personal Motivation

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Abstract

Understanding how emotions stemming from negative outcomes affect investment decisions is critical for studying choice under uncertainty. I build a framework to study how past and anticipated negative emotions interact with an agent's preference and environment to influence her investment level. I show that the dynamic effect of emotions on decisions is more complicated than previously thought and requires a careful analysis of the decision environment to build correct predictions. Using baseball data, I show how to use the theoretical framework empirically to investigate the dynamic impact of emotion and find that it leads to suboptimal pitch velocity decisions.

JEL codes: D90,D91,C13

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1 Introduction

How should people react to adverse outcomes? Economic theory generally posits that reactions should stem from changing beliefs, wealth variations or learning by doing. However, these dynamics do not apply to all contexts. When they are absent, previous events should not affect future investments. Nevertheless, they do. Negative outcomes impact investment in heterogeneous ways when traditional dynamics are absent or controlled for (Heath, 1995; Augenblick, 2016; Dalmia and Filiz-Ozbay, 2021; Martens and Orzen, 2021; Negrini et al., 2022; Guiso et al., 2018).

On the other hand, emotional reactions to adverse outcomes significantly impact people's economic decisions and are a credible dynamic to consider. For example, they prompt individuals to take more risks in various environments (Post et al., 2008; Foellmi et al., 2016; Callen et al., 2014; Passarelli and Tabellini, 2017), while reducing risk tolerance in many others (Cohn et al., 2015; Guiso et al., 2018; Meier, 2022).

The literature has, however, failed to produce consistent results. For example, there is strong evidence that fear triggers pessimistic and anger triggers optimistic appraisals of risky situations (Lerner and Keltner, 2001; Lerner et al., 2015). As such, fear and anger should respectively be associated with less and more risk-taking. However, empirical evidence shows that only fear is consistently associated with less risk-taking (Nguyen and Noussair, 2014; Breaban and Noussair, 2017; Sean Wake and Satpute, 2020; Meier, 2022). In comparison, anger negatively correlates with risk-taking in (Nguyen and Noussair, 2014), shows a weak positive correlation comparable to that of a neutral emotional state in (Breaban and Noussair, 2017). In contrast, (Meier, 2022) measures a positive correlation between anger and risk-taking in survey data.

Given the importance of the influence of such emotions on decisions, it is necessary to understand better why such inconsistencies might arise. However, most economic models that consider emotions in choice under uncertainty do not consider the effect of *past* emotions on current behaviour.[1] This paper participates in the currently small theoretical literature studying the effect of current emotions on future behaviour (Gneezy et al., 2014; Wälde, 2018)

I study the effect of negative emotions stemming from below-expectation outcomes (frustrating events hereafter) on future investment decisions. I focus on a forward-looking

[1] In particular, in these models, emotions only affect behaviour through their expected future effect. See the literature review at the end of this section for more details.

decision-maker who invests resources in projects until she succeeds. Her investment either increases her return or her probability of success. Her emotions are dynamic. When she fails, her emotional level increases with the amount of wasted investment. As such, it accumulates, but it also decays with time. When she succeeds, she gets some utility, called **success utility**, the decision problem stops, and the emotion's effect on utility disappears: success brings emotional relief. The emotion impacts utility in two ways. First, it exerts an **emotional cost**, which is increasing in the emotional level. Second, it triggers **appraisal tendencies** (Lerner and Keltner, 2000, 2001), which distorts the agent's evaluation of her environment and preferences. For example, anger tends to make people more optimistic while fear makes them less (Lerner and Keltner, 2000). In my setting, this translates into a change (positive or negative) in the marginal utility of investment, as emotions change the evaluation of how much utility success would bring.[2]

The conceptual novelty of the model is to define the event that triggers the emotional reaction (frustrating events) while keeping the effect of emotions on utility general.[3] In contrast, most of the current economic literature studies the effect of specific emotions on behaviour. The event-based approach to emotions brings several advantages. (1) Frustrating events provoke various negative emotions, such as disappointment, sadness, fear and anger, which can affect behaviour differently. Restricting the analysis to a single emotion would paint an inaccurate picture of how people react to such situations. (2) It is less culturally dependent than a definition-based approach. Emotions are socio-cultural phenomena. As such, culture will influence how people react to frustrating events and even what type of emotion they can have (Mesquita and Ellsworth, 2001). (3) If one has good reason to do so, the framework can still accommodate specific emotions by restricting how they impact utility and the appraisal of the environment. The main theoretical contribution is to provide a closed-form expression that represents and decomposes the effect of negative emotions on an agent's investment level.[4]

When do negative emotions increase or decrease the agent's investment? On the one hand, the agent wants to limit her exposure to future emotional costs and decrease investment. This is the *emotional cost effect*, which has a negative impact on the investment

[2] Or a change in the evaluation of the marginal return of some production function.

[3] The focus on frustrating events is natural and has a long tradition in psychology (Dollard et al., 1939). In general, modern emotion theories would take the evaluation of an event as frustrating as one of the first steps that could lead to many negative emotions and reactions (Keltner and Lerner, 2010).

[4] For psychologists, this closed-form expression can be considered as the result of the second appraisal in Lazarus and Folkman (1984). In other words, the model endogenously links valence, cognitive appraisal, and appraisal tendencies to action tendencies.

level as long as the negative emotion level increases with investment when she fails. On the other hand, when investment increases her probability of success, the negative emotion also creates an incentive to seek emotional relief. This is the *prospective relief effect*, which always positively impacts investment. Finally, the *appraisal tendency effect* can influence investment either way, depending on the sign of the appraisal tendencies. I show that in a general decision environment, the change in the investment level will be a weighted sum of the three effects.

The characterisation of negative emotion's effect on investment fits and clarifies observations made by psychologists and economists. To continue with the example of fear and anger, psychologists usually associate anger and fear with appraisals of respectively high and low beliefs of control. This translates into a relatively higher willingness to solve the problem triggering the emotion in the case of anger relative to fear. I get this result endogenously. Indeed, the model predicts that an increase in control belief,[5] always leads decision-makers to increase their investment relatively more after a frustrating event, ceteris paribus.[6]

The second contribution is to break down the role of appraisal tendencies in behavioural reactions. I show that positive appraisal tendencies are neither necessary nor sufficient to affect investment positively. As such, while important, appraisal tendencies are not the only force driving an agent's reactions. This explains why anger's (positive) appraisal tendencies do not always translate into more risk-taking behaviour. Indeed, the effect of anger on the investment level will result from a trade-off that can tip in both directions depending on the individual's preferences and the environment. In contrast, the effects of emotional cost and appraisal tendencies go in the same direction in the case of fear, which leads to unambiguous predictions about how it affects behaviour. This effectively breaks the revealed preference (or emotions) argument one can use to infer emotions from actions and explains the discrepancies observed in the literature.

As such, the theoretical framework shows that one must be much more cautious when inferring the behavioural implications of emotions by studying the choice environment and its influence on emotion's dynamic. The last contribution of this paper is to provide such an analysis using Major League Baseball Data. I use a large pitch-by-pitch Major League Baseball Data with a detailed set of controls to do this. These allow to control for most, if not all, possible confounders. I exploit the expected score variations due to the pitch outcome to measure the strength of the negative emotion triggered by failed pitches. I find

[5] Control Bielief is modelled as a higher marginal effect of investment on the probability of success.

[6] That is, they would decrease investment less or increase more.

signs of negative emotion accumulation in pitchers' behaviour. Success seems to dissipate its effect. The emotion tends to increase the pitch's speed. The magnitude of the impact is sizeable. One standard deviation increase in the emotion's intensity[7] accounts for 45.72% to 64.70 % of the magnitude of the effect of one standard deviation increase in fatigue triggered while pitching during an inning (5.16 throws). Finally, I show that sensitivity to negative emotions seems to have implications in terms of productivity. Pitchers whose pitch quality is more affected by negative emotions tend to have lower career average skills measures.

Over the years, many contributions have modelled particular emotions such as regret (Loomes and Sugden, 1982; Bell, 1982), disappointment (Bell, 1985; Loomes and Sugden, 1986) anxiety or excitement (Caplin and Leahy, 2001). In these models, emotions only affect decision-making through the expectation of what can happen in a forward-looking manner, but past emotions do not influence current decisions. In contrast, Loewenstein (2000) develops a framework to show the effect of visceral factors[8] on current decisions but does not consider their dynamic. Here, both past and anticipated emotions affect decisions, allowing novel theoretical predictions. Two exceptions, which consider the dynamics of emotions, are (Wälde, 2018) and (Gneezy et al., 2014), which model the dynamic effect of stress and guilt on behaviour. Finally, Dillenberger and Rozen (2015) axiomatise a model of history-dependent risk-attitude. The paper shares similarities to this one, as risk aversion depends on the history of the realisation of lotteries, but the intensity of past frustrating events does not play a role in determining risk aversion. I would also like to mention an active literature modelling emotions in interpersonal contexts using psychological game theory tools (Battigalli and Dufwenberg, 2007, 2009; Battigalli et al., 2019).

Section 2, presents the general framework and explains the psychology behind the modelling choices. In section 3, I study the determinant of emotional reactions stemming from frustrating events. I first focus on the impact of appraisal tendencies. I then study how frustration interacts with the agent's control over her success probability through investment. Section 4 investigates the effect of frustration accumulation on pitcher's behaviour using pitch-by-pitch Major League Baseball data. Section 5 concludes and discusses the forward-looking and time consistency assumption used in the model.

[7]Here, intensity is proxied through the intensity of the frustrating event - the gap between what was expected and what happened. A standard deviation change in intensity equals 0.27 expected runs the opposing team could score because of the pitch's failure.

[8]Visceral factors are drive states that have a hedonic impact on utility, such as hunger and possibly emotions.

2 General Framework and Psychological Principles

The general decision environment describes an agent investing resources to solve a problem until she succeeds. Although stylised, it allows for clean predictions and is an ideal setup for studying negative emotion accumulation and its effect on behaviour. The modelling principles described in this section are simple, portable, and easily adapted to fit other decision environments.

Decision environment

I study an agent who is investing resources $x_t \in \mathbb{R}_+$ at every point in time $t \in \mathbb{R}_+$, to possibly get a successful investment.[9] Investment might increase utility in case of success, or increase the probability of success, depending on the application in the following sections. Success is determined by a Poisson process q, where $q = 0$ if the investment failed. The probability that q increases by 1 during dt, i.e. $dq = 1$, is given by $\bar{\pi}(x_t)dt$, where $\bar{\pi}(x_t)$ is the arrival rate of the Poisson process during the time interval dt. In other words, $\bar{\pi}(x_t)dt$ denotes the probability of getting a success during the time interval dt. As such, the probability of failing from 0 to time t, denoted p_t, is given by an inhomogeneous Poisson point process with:

$$p_t = \frac{\left(\int_0^t \bar{\pi}(x_s)ds\right)^0}{0!} e^{-\int_0^t \bar{\pi}(x_s)ds} = e^{-\int_0^t \bar{\pi}(x_s)ds}. \tag{1}$$

As such, p_t takes a convenient form as it will act as an endogenous discount rate to the inter-temporal maximisation problem.

Psychological dynamics

In what follows, I use the umbrella term **frustration** to name all emotions stemming from frustrating events.[10] Frustration *should not* be interpreted as a specific emotion and must be understood as a theoretical concept. Interested and specialised readers can refer to Supplemental Appendix E for more information about the frustration concept.

[9]Resources can represent many different things: money, capital, time, force, space, effort etc..

[10]The most famous use of the term frustration comes from the work on the frustration-aggression hypothesis Dollard et al. (1939). However, they use the term frustration to denote frustrating events. I use the term frustrating event to denote the event triggering frustration. See Breuer and Elson (2017) for a modern summary of the theory.

Emotions are dynamic by nature. The central part of the framework is that frustration f_t is modelled as a stock that lingers if the investor continues to suffer failures. Specifically, frustration increases by the amount of wasted investment when she fails but decays at a rate $\delta \in (0, 1)$:

$$\dot{f}_t = x_t - \delta f_t \tag{2}$$

Where $\dot{f}_t$ represents the derivative of frustration with regards to time. The decision problem stops if the investor succeeds at time τ and all future utilities equal 0. These modelling choices capture several dynamic features of emotions. First, the decay rate δ represents that emotional response tends to fade when time passes and revert to a neutral state (Elster, 1998; Loewenstein, 2000; Lerner et al., 2015).

Second, the effect of frustration disappears after achieving success. This represents the "*goal-attainment hypothesis*", whereby the impact of emotion on behaviour ceases when the problem at its source is solved (Han et al., 2007; Goldberg et al., 1999). This modelling choice will be especially relevant when investors cannot do precise "accounting" of the loss and cannot recuperate it. For example, when individuals fail when spending time or exerting effort, investment is hard to measure and impossible to get back. Alternatively, when investing in a project that is important to the decision-maker, emotional relief might only be attainable when the project comes to fruition and not when the investors break even, as the financial return was not what was important to begin with.

Finally, frustration accumulates. Emotions of the same sign (positive or negative) that are triggered consecutively tend to overlap and create a phenomenon called "*emotion augmentation*" (Pe and Kuppens, 2012; Kuppens and Verduyn, 2017). In these cases, the effect of one emotional experience tends to increase the following one. I represent this with a time dependence between current and past frustration levels.

Note that frustration could also increase independently of the investment level by a discrete increment. I investigate this case in supplemental appendix B.

Material preferences

Investment is costly. I represent the investment cost as an increasing and (weakly) convex cost function $c(x)$. If she succeeds, the agent gets utility $u(x_t, f_t)$, called **success utility**, which is increasing in investment and jointly concave in investment and frustration. The impact of frustration on success utility is described in the following subsection.

The impact of frustration on preferences

I consider that frustration impacts utility in two ways. First, emotions stemming from frustrating events will be negative emotions. As such, if she is still investing at time t, the agent suffers an emotional cost stemming from her frustration stock. I represent the emotional cost as an increasing and convex function $v(f)$. Psychologists would call the negative effect of $v(f)$ on utility the *valence* of the emotion. In other words, $v(f)$'s construction reflects that ceteris paribus, people prefer to be less afraid or less angry and prefer situations with lower f.

Second, frustration triggers *appraisal tendencies* that affect the agent's judgement. As such, some options can become more appealing when in an emotional state than in a neutral state. Appraisal tendencies are important as emotions with the same valence can have different appraisal tendencies (e.g. fear and anger). Similarly, emotions with opposite valence can have the same appraisal tendencies (e.g., anger and happiness) (Lerner and Keltner, 2001, 2000). To represent this, I say that frustration exhibits **negative (positive) appraisal tendencies** if $u_{xf} < (>) 0$.[11] In other words, negative (positive) appraisal tendencies decrease (increase) the marginal utility of investment one gets in case of success. For example, positive appraisal tendencies characterise a situation where the decision-maker believes her actions are becoming more efficient at increasing her success utility or simply because she marginally values success more.[12]

The inter-temporal decision problem

Let me now give the general form of the instantaneous expected utility representing the decision maker's preferences:

$$U(x_t, f_t) = \bar{\pi}(x_t)u(x_t, f_t) - v(f_t) - c(x_t) \tag{3}$$

During a time interval dt, the probability of success is given by $\bar{\pi}(x_t)$. The agent gets her success utility $u(x_t, f_t)$ when she succeeds. Note that in this general formulation, the probability of success and utility depend on x_t. While I do not focus on this general case in the next sections, it is not improbable.[13] Finally, the two last terms of equation (3)

[11] In what follows, I represent derivatives by subscripts: $f_x = \frac{\partial f(x,y)}{\partial x}$, $f_{xy} = \frac{\partial^2 f(x,y)}{\partial x \partial y}$, $f_{xx} = \frac{\partial^2 f(x,y)}{\partial x^2}$.

[12] Appraisal tendencies could affect motivation through another channel: the decision maker's actual or assessment of the Poisson law arrival rate, as emotions influence risk assessment (Lerner and Keltner, 2000, 2001). This framework can easily be adapted to model these effects.

[13] A researcher increasing her effort level would increase the probability (higher $\bar{\pi}$) and the quality of a

represent the emotional cost $v(f_t)$ the agent has to endure because she is still investing at time t and the investment cost $c(x_t)$. Let $\rho > 0$ be the agent's subjective discount rate. As the decision-making process only continues until time t with probability p_t, the agent must also discount these situations accordingly. Let $\phi(x_t, t) = \int_0^t \bar{\pi}(x_s)ds + \rho t$ denote the effective discount rate. The inter-temporal decision problem is given by:

$$V(t_0, f_{t_0}) = \max_{x_t \in \mathbb{R}_+} \int_{t_0}^{\infty} e^{-\phi(x_t,t)} U(x_t, f_t) dt \tag{G.O}$$
$$\dot{f}_t = x_t - \delta f_t$$
$$f_{t_0} \text{ given}$$

As such, the decision maker, who stands at t_0 decides on optimal investment levels x_t for all t. The function x_t, $x_t : [t_0, \infty) \to \mathbb{R}_+$ gives, for every t, the optimal investment level the agent is willing to put in if she is still failing at time t.[14] To ensure sufficiency of the first order conditions, I assume joint concavity of $e^{-\phi(x_t,t)}U(x_t, f_t)$. This assumption is not trivial, as the endogenous probability of success can create important convexities in the inter-temporal maximisation problem. I relax the concavity assumption in supplemental appendix C and show that the model transforms into a rational addiction model Becker and Murphy (1988).[15]

I focus the analysis on two distinct set-ups. In section 3.1, I explore the inter-temporal effects of appraisal tendencies because they are reminiscent of previous works in the literature (Loewenstein, 2000; Laibson, 2001).[16] In Section 3.2, I consider an environment where investment affects the probability of success with $\bar{\pi}(x) = \gamma \cdot x$, $\gamma > 0$, but frustration does not directly influence the preferences for the choices at hand i.e. there are no appraisal tendencies. The general model is solved in supplemental appendix A.1.

publication (higher u). I solve the model for the general case in supplemental appendix A.1.

[14]I focus on situations where frustration accumulation is the only dynamic. In supplemental appendix D, I explore the combined effect of learning-by-doing and frustration accumulation and characterise the conditions under which the optimal investment path can become non-monotonic.

[15]I show that the decision-maker can become rationally addicted even when $U_{xf} = 0$, that is, without any habit formation process, which are the usual drivers of addictive behaviours in traditional rational addiction models.

[16]Indeed, appraisal tendencies affect the agent's marginal utility in case of success, as do Laibson (2001) 's cues (which create craving/disgust and then habits) or Loewenstein (2000)'s visceral factors.

3 The formation of emotional reactions

3.1 The emotional cost and Appraisal tendencies

To get cleaner dynamics and isolate the role of appraisal tendencies, I focus on situations where the success probability is exogenous, i.e. the arrival of the Poisson process will be $\bar{\pi}(x_t) = \pi > 0$. As such, the investor invests because her success utility increases with investment. The instantaneous expected utility at time t for a period dt is:

$$U(x_t, f_t) = \pi u(x_t, f_t) - v(f_t) - c(x_t) \tag{4}$$

The overall optimisation problem is as in (G.O) presented before, with the specification given here.[17] After linearising the system obtained by solving the optimisation problem around the steady state, I get the following system:[18]

$$\dot{\hat{x}}_t = (\rho + \pi + \delta)\hat{x}_t + \Omega_A^* \cdot \hat{f}_t \tag{5a}$$

$$\dot{\hat{f}}_t = \hat{x}_t - \delta \hat{f}_t \tag{5b}$$

$$\text{Where} \qquad \Omega_A^* = \frac{1}{U_{xx}^*}\left(U_{ff}^* + (\rho + \pi + 2\delta)U_{xf}^*\right) \tag{6}$$

Where U_{ff}^*, U_{xf}^* and U_{xx}^* indicate the value of the second-order derivatives at the steady state. The first line in (5a) is the Euler equation; it indicates how investment varies for a given level of investment $\hat{x}_t$ and $\hat{f}_t$.(5b) reiterates the frustration law of motion.

The variable of interest is Ω_A^*, which captures the temporal complementarity of investment and frustration when there are appraisal tendencies.[19] Its sign indicates whether an

[17] One can ensure that the solutions are in the interior of the control variable set by setting Inada type conditions $\lim_{x\to 0} U_x(x, f) = \infty$ and $\lim_{x\to I} U_x(x, f) = -\infty$ for all f. To have sufficiency, the Hamiltonian must be jointly concave in x and f. In other words, the appraisal tendencies of frustration cannot be too strong. Finally, note that these conditions also ensure the uniqueness of the steady-state

[18] I introduce a variable transformation such that the steady-state value of frustration and investment equals zero. As such, values below the steady state level are strictly negative. The scaled variables are denoted with a hat.

[19] To be precise, Ω_A^* is the value of the Volterra derivative of the optimal investment path functional $X^*(t)$ at the steady state, see Ryder Harl E. and Heal (1973), for more details. It is also easy to notice the similarity with Becker and Murphy (1988) Rational Addiction model because the appraisal tendencies U_{xf}^* are akin to addictive tendencies. In the next section, I set these appraisal tendencies to zero to study different dynamics. In Appendix C, I show that I can get Rational Addiction dynamics without appraisal tendencies.

increase in frustration today increases or decreases the optimal investment level tomorrow. Proposition 1 formally shows the dependence of the system's dynamics on Ω_A^* and clarifies when the maximisation problem is well-behaved and has a stable steady-state.

Proposition 1. *1. An increase in f leads to an increase (decrease) in investment provision x if and only if $\Omega_A^* < (>) 0$,*

2. The system exhibits saddle path stability as long as $\Omega_A^* > -\delta(\rho + \pi + \delta)$.

Proposition 1 and the Euler equation (5a) give us a clear interpretation of Ω_A^*. Frustration's impact on the investment level will be determined by the value and sign of Ω_A^*. Studying equation (6), we see that the behavioural reaction will have two components (1) an *emotional cost effect* $U_{ff}^* < 0$, which represents the effect of the frustration cost on behaviour. The increasing marginal cost of frustration leads to steeper utility losses if frustration increases. As such, the agent must decrease investment to equate marginal utility to the increased shadow cost of frustration.[20]

The second component is the *appraisal tendency effect*, given by U_{xf}. The appraisal tendency effect measures the change of the marginal utility when frustration changes. They naturally translate into a change in the optimal investment level. If positive, they can mitigate the negative effect of the emotional cost on investment. If strong enough, they can trigger an increase in investment.

This leads to my first observation: although appraisal tendencies can amplify or mitigate the emotional cost effect, positive appraisal tendencies are not sufficient to have agents increase their investment level.

Also note that the parameters ρ, π and δ play an essential role. Suppose the investor does not discount future failures heavily because she thinks that failures are likely (low π). In that case, the effects of appraisal tendencies are relatively discounted compared to long-term frustration costs because they only affect success utility. Similarly, lower time-discounting (low ρ) would put relatively more weight on U_{ff}^*. The opposite effect happens for the decay rate of frustration, which amplifies the effects of appraisal tendencies. If the decay rate increases, frustration cannot reach higher levels and is potentially less emotionally costly in the long run. This shows that emotional reaction balances long-term implications: the expected emotional cost of continuing to fail versus the short-term benefit of achieving success.

[20] Note that if frustration increases in exogenous increments, as in supplemental appendix B, the emotional cost effect does not affect investment.

3.2 The effect of the controllability of the environment

This section shows how the perceived control of an environment influences emotional reactions stemming from frustrating events. Perceived control is generally seen as a critical driver in emotional responses. People who believe they control their environment more also tend to be more proactive in solving the problem causing their distress (Smith and Ellsworth, 1985; Lerner and Keltner, 2001). It is also one of the main features differentiating situations triggering fear and anger. One appealing feature of this framework is that its solution provides action tendencies (increase or decrease investment) that resemble the reaction of anger and fear, depending on how they control their environment.

Situations where decision-makers control the probability of outcomes are essential in many applications, such as principal-agent relations or self-protection frameworks. I equate control over the environment with the control over the probability of success during each time interval dt, given by $\bar{\pi}(x_t) = \gamma \cdot x_t$, with $\gamma > 0$. The higher the γ, the higher the control. Let $\phi(x_t, t) = \int_0^t \gamma x_s ds + \rho t$. Let me use the following specification to fix ideas.

$$U(x_t, f_t) = \gamma \cdot x_t u - v(f_t) - c(x_t) \tag{7}$$

Success utility $u > 0$ is fixed, and the probability of having a success during dt, $\gamma \cdot x_t \cdot dt$, depends on x. Notice that $U_{xf} = 0$: there are no appraisal tendencies, and changing the frustration level does not change the marginal value of investment x_t at time t. However, this does not mean that frustration does not affect the inter-temporal marginal value of investment. Indeed, the relevant objective function to consider at time t is $F(x_t, f_t) = e^{-\phi(x_t,t)}U(x_t, f_t)$. The effect of x_t is inter-temporal by nature as an increase in x_t today changes the probability of continuing tomorrow. Even though today's frustration cost is sunk, as x_t does not affect $v(f_t)$, it affects the probability of still being frustrated tomorrow. Frustration impacts the marginal utility of investment because $F_{xf} > 0$. Solving the optimization problem G.O given the specification described here, (see supplemental appendix A.1) yields the following Proposition:

Proposition 2. *Let* $\Omega_C^* = \frac{1}{U_{xx}} \left(U_{ff}^* - (\rho + \gamma \cdot x^* + 2\delta) \frac{\gamma U_f^*}{\rho + \gamma \cdot x^* + \delta} \right)$.

1. *An increase in frustration leads to an increase (decrease) in investment provision if and only if* $\Omega_C^* < (>) 0$,

2. *The system exhibits saddle path stability as long as* $\Omega_C^* > -\delta(\rho + \gamma \cdot x^* + \delta)$.

Ω_C^* and Ω_A^* share the same structure. On the one hand, the potential future emotional cost U_{ff}^* remains. On the other hand, the temporal complementarity between frustration and investment is now composed of another factor: the *prospective emotional relief effect*. It represents how effective additional effort is at avoiding the additional emotional cost a new failure would entail. More precisely, it is composed of the shadow cost of frustration at the steady state, representing the marginal utility cost of increasing frustration multiplied by γ, the marginal effect of investment on the arrival rate of success. As such, the prospective emotional relief effect depends on how effective investing is at increasing the odds of success and how much the potential increase in frustration will decrease utility.

The fact that the prospective emotional relief effect is multiplied by $(\rho+\gamma\cdot x^*+2\delta)$ shows that the same long versus short-term emotional trade-off exist in Ω_C^* and Ω_A^*. In general, the sign of Ω_C^* will determine the system's dynamics, and the decision-maker will face a similar trade-off as with Ω_A^*. As such, I will not spend time on it. However, contrary to the previous section, where appraisal tendencies could influence investment either way, environment controllability can only increase the investment level when frustration rises. This aligns with the psychological literature (Smith and Ellsworth, 1985; Lerner and Keltner, 2001).

3.3 Practical Implications of the framework

Hypothesis Testing: The fact that an emotional trade-off can exist between the emotional cost, the prospective emotional relief and the appraisal tendency effect has important implications for the study of emotions. Research in economics and psychology often uses discrete emotion types to study their impact on behaviour, such as the effect of anger or fear on certain behaviours. This section shows that attaching a typical action to an emotion can lead to misleading behavioural implications. Positive appraisal tendencies and a high control belief are associated with anger, and these two factors positively affect the change in the investment level, which fits insights from the psychological literature. However, the two factors' influence will only result in an actual increase in the investment level when the emotional cost effect is relatively low enough. As such, automatically associating anger with an increase in investment to solve a situation might be misleading and lead to incorrect experimental hypotheses. The effect of anger on investment levels comes from a trade-off that can go in either direction, depending on the person and the situation. On the other hand, the effects of emotional cost and appraisal tendencies align in the case of fear, which allows for clearer predictions about how it influences behaviour. This effectively rationalises the empirical pattern of the literature showing that fear's appraisal tendency systematically translated into behavioural implications while anger's one did not. **Emotion revealed?** Standard economic datasets have, in general, information about what people do or what they buy. The fact that anger and sadness are sometimes behaviourally equivalent can also be problematic when trying to infer which emotion was triggered by an event. As such, one cannot make a one-to-one revealed preference argument about behaviour and implied emotions.

The decision environment matters. Emotional reactions are triggered by the appraisal of the environment. This is good news for experimentalists as they effectively control the decision environment and, consequently, the appraisal of it. As such, it is relatively straightforward to develop an experimental hypothesis by varying the environment. For example, in simpler two-period experiments one can vary features such as γ, π, or the decision timing for the decay rate.

The next section studies how one can approach emotions' dynamics to infer valuable information about how they will impact behaviour on non-experimental data. This starts with studying the decision environment and the effect of the decision-maker's action on the frustration law of motion and its stochastic structure.

4 Illustration

This part aims to provide an illustration that fits the model's set-up and test its building blocks. It also shows how one can use an event-based approach to emotion to get meaningful information about the emotional process to which individuals respond. To do so, I use Major League Baseball pitching data to study the effect of frustration accumulation on pitcher behaviour. *Pitchers*, are part of the defending team and throw the baseball at another teammate, called the *catcher*. Between these two players is a third player called the *batter*, who is in the offensive team and tries to score points. The pitcher aims to prevent the batter from scoring points by pitching balls that are hard for the batter to hit.

The primary dependent variable for the analysis is pitch velocity, arguably the variable over which the pitcher has the most control and which can be considered a natural measure of effort (or investment). Moreover, it has a natural interpretation. Faster pitches should increase the odds of success until the pitcher loses precision. As I argue below, pitchers should always throw their pitches at an optimum speed, given a pitch type and a strategic environment. As such, one can interpret any significant frustration coefficient as over- or under-investment relative to the optimal level with the appropriate control strategy.

Isn't Baseball a strategic game? It is true that changing the speed of the pitch might be a strategy to surprise the batter. However, baseball culture has developed so that pitches with significant differences in speed are also categorised differently because the catcher and the pitcher must communicate them in sign language.[21] Each category is called a pitch type and differs in terms of speed, variations in release point, spin, and mechanics that affect the ball's velocity. For example, the slow version of a fast ball is called a change-up and is, as such, categorised differently. Much of a pitcher's strategy is expressed through the strategic randomisation of the type of pitch he is throwing to catch the batter off-guard. From a game theoretic point of view, a pitcher's optimal strategy should be a probability distribution over the pitches in his repertoire, which is a mixed Nash equilibrium. As my model does not consider such strategic considerations, I control for the different pitch types and eliminate this strategic component of the game from my analysis. In other words, my analysis exploits the fact that, within pitch type, speed should not be affected by strategic consideration.[22]

Objective of the analysis

The illustration shows one can proceed to carry out an empirical analysis using an event-based approach to emotion. I structure this approach around three axes. The first axis evaluates the rational incentive structure surrounding pitch velocity choices. Given baseball rules, it essentially determines the pitcher's objectives and establishes a control strategy to control for other factors that can influence the pitch speed. These include pitch type, strategic considerations, bayesian updating about one's ability, other emotional processes etc...

The second axis identifies the emotional incentive structure surrounding pitch velocity choices. If first defines what can be considered a failure in baseball, and how frustration is likely to accumulate. Next, I also measure the implication of changing pitch velocity on the probability of success, given that it is a key element of how people would react to frustration. This analysis then allows us to understand exactly which effect (appraisal tendency, emotional cost and prospective emotional relief effect) is at play.

The third axis estimates the effect of frustration accumulation on pitcher behaviour. I then explore the features of frustration dynamics and how they relate to the building block

[21] Catchers must be prepared for the type of pitch the pitcher is throwing, otherwise it would be too hard to catch. Using signs, the catcher proposes a pitch type that the pitcher should throw and prepares accordingly to catch it.

[22] Although I carefully control for possible additional confounders.

of the model. I focus on frustration's temporal effect and the prospective emotional relief effect. Finally, I also explore the relation between frustration sensitivity (or whether pitchers react to frustration) and their average career performance in supplemental appendix F.3.

The dataset

The data set consists of measurements for every pitch thrown during the 2010-2019 period retrieved on a Major League Baseball (MLB) run website for more than 7 million pitches. Physical information about the pitches is gathered through the PITCHf/x system for the 2010-2015 period and Statcast for 2015-2019. These two systems are automated camera systems developed to analyse player's movements. Physical information about the pitches includes speed at release, location of the pitch at different points in time, spin rate, spin angle, and acceleration. The dataset includes detailed game information, including game, pitcher and batter identifiers, base occupancy, as well as strike, ball and out counts. The MLB also provides information about the batter team's change in run expectancy before and after each pitch.

4.1 Baseball Rules and Pitchers' objective

I first give an extremely summarised version of Baseball rules. The basic unit of play is called an inning, and a game typically includes nine innings. An inning comprises two half-inning periods where attacking and defending teams switch roles. There are nine players in each team. The game takes place in a field with four white rectangle bases forming a square.

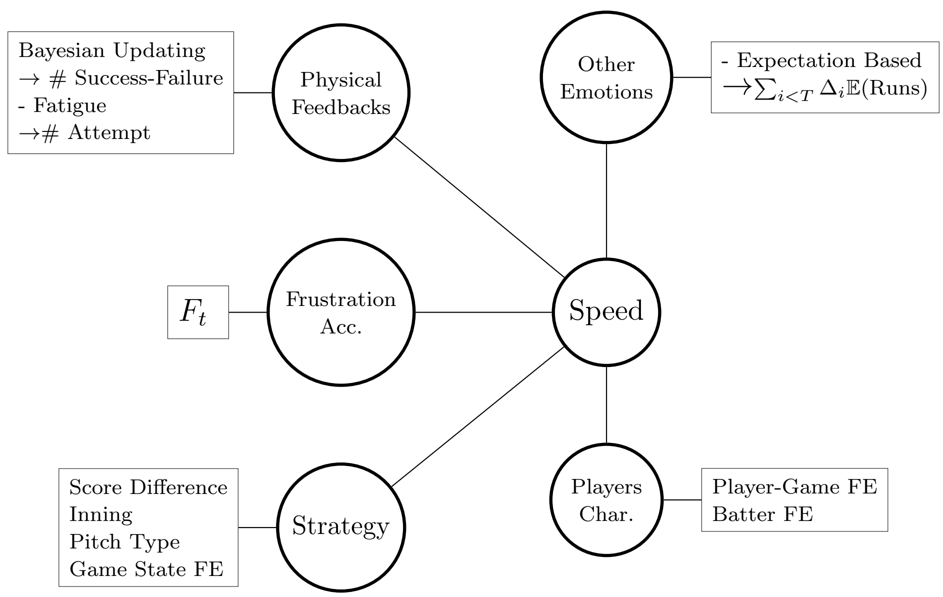
The batter, who is in the offensive team, aims to hit the ball to advance as far as possible around four bases while the defending team tries to get the ball back to the catcher. If the batter does not reach the fourth base (home plate), he waits on one of the bases for the next opportunity to advance during the following plays. A base with a player on it is "loaded". Once a batter is out or on a base, one of his teammates replaces him, and the process starts again. If one offensive team member goes around the four bases, the team scores a run (a point). The team with the most runs at the end of the game wins.

A pitcher's goal is to prevent the other team from scoring runs. He does this by throwing pitches that are difficult for the batter to hit and to get batters out as soon as possible. Indeed, when three batters are out, the half-inning ends, thus preventing the other team from scoring further runs. The three more important pitch outcomes are (1) balls: if the pitch is not thrown correctly, it is considered a ball, and four balls grant first base to the batter. (2) strikes, if the batter did not hit a valid pitch in a valid area, the batter is out with three strikes;[23] (3) the batter bats the pitch in a valid area: it is *in game*.

4.2 Estimation Strategy

Even though it is impossible to develop a quasi-experiment in this dataset, the sheer amount of information available for every pitch makes it possible to control for most confounders that could bias the analysis. Before investigating the potential effect of frustration on speed, one must review all the possible reasons why a pitcher might vary the speed of a given pitch. Figure 1 gives a schematic view of the possible influences one must control and Table 1 describes the corresponding controls used in the regressions.

Figure 1: Schematics of factors that can influence pitch velocity



Note: Schematic view of the possible confounders of the analysis. Note that some elements could also be in other categories, and the links between categories are not represented. I mention these overlaps between categories in the discussion below. $\sum_{i<T} \Delta_i \mathbb{E}(\text{Runs})$ represents the difference in run expectancy between the current throw and the beginning of the half-inning.

Strategy: Much of a pitcher's strategy is expressed through the strategic randomisation of the type of pitch he is throwing to catch the batter off-guard. To control for this,

[23]A pitch batted in a non-valid area is called a foul.

I use MLB's pitch type classification.[24] Other strategic interactions might also affect the pitches' speed. Suppose a game or plate appearance is not particularly competitive. In that case, the pitcher might decide to reduce the speed of his pitches to save his arm. The *Game State* fixed effect is a categorical variable that tries to describe the current state of an inning comprehensively. The objective is to capture Game-State dependent strategic considerations affecting the speed of a given throw. Each category is a 6-digit number, representing the number of outs, strikes and balls, whether a player is on first, second or third base.[25] Finally, I also control for Batter and Pitcher × Game fixed effect, inning number and difference in score to account for the plausible strategic impact on player's behaviours.

Physical feedback: Belief updating might play an essential role in a pitcher's pitch velocity decision. First, a player might update his beliefs about his current ability by looking at the performance of his throws. As such, I create a variable counting the number of successes and failures. Assuming that the agents consistently update failures and successes, this should proxy the effect of Bayesian updating. It also allows for controlling for asymmetric updating where the agent puts relatively more weight on bad news, for example. I also control for the number of throws the pitcher already threw in an inning to consider fatigue.

Other emotions: The major reason why one cannot exploit some quasi-experimental design with this dataset is that there are no neutral events in terms of emotion. In particular, a failure will be frustrating and a success elating. As such, controlling the effect of emotions stemming from elating events is important. Emotional triggers are usually belief dependent (Keltner and Lerner, 2010). One natural moment to consider in the belief distribution is the expectation. To control for other possible emotions, I include Δ Exp., which represents the difference in the opposing team's point expectation between the current throw and the beginning of the half-inning. The variable is computed by the MLB.[26] As such, Δ Exp. together with Bayesian updating, should capture the effect of other belief-based emotions.

Player Characteristics: Sometimes, physical characteristics can influence pitches'

[24]They use a player-specific neural network called PitchNet to classify pitches. The accuracy reaches 78.30% for rookies and 96,25% for regular players. See Sharpe and Schwartz (2020).

[25]For example, a Game State variable being equal to 213101 describes an inning with two outs, one strike, three balls, one player on first base, no player on second and one on the third.

[26]Computing the expected number of runs for any game state is relatively straightforward as it is simply the historical average number of runs that are scored until the end of the innings given the current game state.

velocity. Temperature, weather, or a pitcher's current condition can significantly impact a pitcher's performance. However, it should also be easily controlled with the Pitcher × Games fixed effect.

Table 1: Controls for regressions

Possible factors	Controls
Fatigue	*# Attempt*: Number of pitches in Inning
Player's current ability	*Game × Player FE*
Strategy on a given game	*Game × Player FE*
Strategy in a Game State	*Game State*: #Out × #Strikes × #Balls × Bases load
Batter characteristics	*Batter FE*
Pitch type	P_{type}: Pitch type FE
Score	*Difference in Score*
Score in inning	Δ *Inscore*: Difference in score in the inning.
Inning	*Inning FE*
Bayesian Updating	*# Failures and Successes*
Other expectation-based emotions	Δ *Exp.*: Net Difference in exp. since start of Inning

4.3 Failures and the frustration accumulation process

I now analyse the emotional incentives surrounding pitch velocity decisions and how to measure frustration. Let R_t be the score expectation of the opposing team during this half-inning right before throw number t, where $t \leq T$ and T are the numbers of consecutive throws the pitcher does during this half-inning. Given that the pitcher's objective is to prevent the other team from scoring runs, he always wants to have $R_{t+1} - R_t < 0$, where $runs_t$ indicates the number of runs the other team scored at time t. I can then classify pitches' outcomes by their effect on the expected score. This yields the following definition of failure:

Definition. *A pitch a time t is a failure if it increases the run expectancy of the opposing team, that is, if $R_{t+1} - R_t \geq 0$.*

Now, I need to measure the level of frustration associated with any failure in the game. I consider the difference in run expectancy before and after a failure as my frustration proxy. Doing this allows me to get two desirable properties. First, the measure is in runs and is easily interpretable. Second, it objectively measures the frustration intensity, as different failures can have other repercussions. For example, a ball (an invalid throw) at the beginning of an inning will have a negligible impact on the expected game score, while a home run or a fourth ball with three bases loaded can be decisive. [27] Accordingly, the measure reflects that minor mistakes are less frustrating and that the same event can trigger different levels of frustration depending on the state of the game[28].

Definition. *The frustration triggered by a failure at time t, Δf_t is:*

$$\Delta f_t = R_{t+1} - R_t \geq 0 \tag{8}$$

Next,[29] following the model's set-up, the frustration level increases by the (positive) difference in run expectancy in case of a frustrating event and goes to 0 following success. Let C_t represent the set of time period s that are in a string of consecutive failure before t, that is:

$$C_t = \{s < t : \Delta f_s \geq 0, \text{ and} \forall s' : \, t > s' \geq s \, : \Delta f_{s'} \geq 0\}$$

Definition. *The stock of frustration at the beginning of period $t \leq T$, F_t is:*

$$F_t = \sum_{s \in C_t} \Delta f_s$$

As such, in case of consecutive failures, the measured frustration law of motion is:

$$F_{t+1} = \Delta f_t + F_t, \tag{9}$$

while $F_{t+1} = 0$ if there is a success at time t, as C_{t+1} is empty.

Note that the frustration does not directly depend on the pitcher's velocity decision but only on the consequences of those actions and the state of the game. As such, the frustration increase in case of failure do not depend on the pitch's speed. In other words, the frustration law of motion is a frustration accumulation process with exogenous increments, as in supplemental appendix B. In particular, it means that the emotional cost effect will

[27] Indeed, some baseball fans would even be reluctant to consider a ball at the beginning of an inning as frustrating. A ball at the beginning of an inning would increase frustration by 0,034. This is negligible but still worse than any other non-frustrating outcome (strikes, fouls, ...)

[28] Notice that the measure can sometimes miss subtleties specific to the throw. For example, it is possible to have a positive difference in run expectancy after flawlessly executed pitches where the rest of the team did not live up to the pitcher's performance or for failures where the pitcher was at fault. In other words, this measure only takes the effect of the pitch on the game's outcome into account without characterising who is responsible for the poor performance. However, the measure should be a good proxy for the frustration triggered.

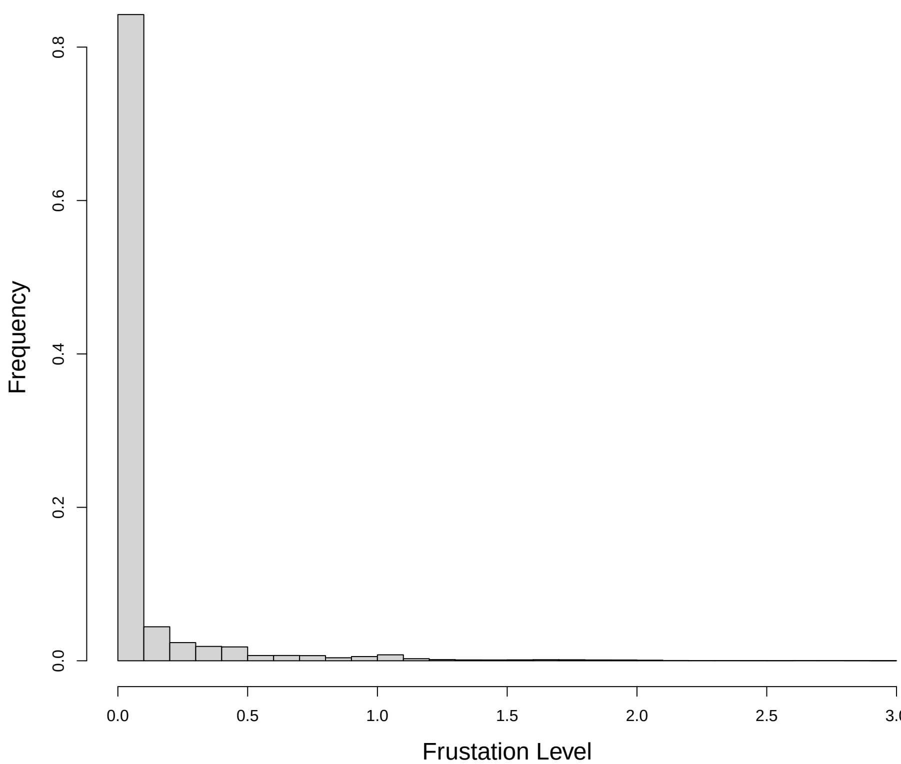
[29] The previous definition also considers a foul ball after two strikes as a failure, but with a frustration value of 0. As such, it is a neutral event that does not break strings of consecutive failures.

not affect how the pitcher will react to frustration, as it is unlikely that he will feel more or less frustrated when throwing harder. I summarise this in the following remark:

Remark. *The pitcher's frustration increases with exogenous increments that do not depend on the speed of the pitches. As such, the emotional cost effect does not affect the velocity of pitches.*

Moreover, this *measured* law of motion does not feature a decay rate δ, while the *actual* law of motion does. I will get back to this when studying frustration's temporal effect in section 4.6. Figure 2 gives insight into how the frustration stock is distributed in the dataset.

Figure 2: Frustration level histogram



Frustration stock distribution. I do not represent observation with frustration levels higher than 3 (less than 0.05% of the observations).

Having a high frustration stock is not common. This is natural. First, the definition chosen is somewhat extreme, as frustration goes to 0 after any success. An alternative could be to have only a share of F_t remaining after a successful pitch. However, determining the level of the share seems arbitrary. I prefer to show that the results hold under the most extreme assumption, which would underestimate the effect of frustration. Second, frustration is measured in runs. As such, it seems natural that professional pitchers do not concede high numbers of consecutive runs without being replaced.[30]

4.4 Pitch velocity's effect on the success probability around the optimum

Understanding how pitch velocity influences the probability of success is important to understand whether the prospective emotional relief effect will influence velocity decisions. Intuitively, faster pitches are more challenging to hit, so speed should positively affect the odds of success and the chance of getting emotional relief. On the other hand, faster pitches are also harder to control, and less precise. To settle this question, the following regression estimates the impact of speed on the probability of success.

Table 2 presents the result of an OLS regressing the speed and the log of the speed of pitches on the probability of success. Speed has a statistically significant but economically small effect on the likelihood of success. Increasing the release speed by one per cent increases the chance of success by less than 0.0014%. To compare this to natural speed variations in the data, one can consider the average game and pitcher-specific standard deviation in speed at the inning level. Table F.1.1 in supplemental appendix F.1 presents this standard deviation for different pitch types. The standard deviation is, in general, around 1. As such, according to columns (3) and (4), a pitcher would need to increase their pitch speed by five standard deviations to increase their probability of success by a negligible 1%. Of course, this estimate should be considered as a local effect, as pitchers cannot significantly increase their pitches' speed without losing precision or risking injury.

Given the low adjusted R^2, velocity does not seem to be a credible explanatory factor to predict success. Overall, Table 2 results are not surprising. MLB pitcher's market is competitive, making it hard to believe that a pitcher would not throw at the best of their capabilities given the strategic environment.

Moreover, supplemental appendix F.2 shows a regression result measuring how frustra-

[30]Note that a high frustration level is not necessarily highly correlated with a high probability of the pitcher getting replaced. This is because frustration goes to 0 after a success. Also, a similar increase in frustration can happen over one or many failures.

Table 2: Effect of Speed on probability of success

Dependent Variable:	P(Success)			
Model:	(1)	(2)	(3)	(4)
Variables				
Speed	0.1463***	0.1463***	0.0020***	0.0020***
	(0.0283)	(0.0097)	(0.0003)	(0.0001)
# Failures	0.0193***	0.0193***	0.0193***	0.0193***
	(0.0006)	(0.0002)	(0.0006)	(0.0002)
# Successes	-0.0176***	-0.0176***	-0.0177***	-0.0177***
	(0.0007)	(0.0002)	(0.0007)	(0.0002)
Attempt	0.0058***	0.0058***	0.0058***	0.0058***
	(0.0004)	(8.77×10^{-5})	(0.0004)	(8.78×10^{-5})
Δ Exp.	-0.0209***	-0.0209***	-0.0210***	-0.0210***
	(0.0027)	(0.0015)	(0.0027)	(0.0015)
Δ Score	-0.0050***	-0.0050***	-0.0050***	-0.0050***
	(0.0005)	(0.0003)	(0.0005)	(0.0003)
Δ Inning Score	0.0081***	0.0081***	0.0081***	0.0081***
	(0.0027)	(0.0015)	(0.0027)	(0.0015)
Fixed-effects				
P_{type}	Yes	Yes	Yes	Yes
Game State	Yes	Yes	Yes	Yes
Player × Game	Yes	Yes	Yes	Yes
Batter	Yes	Yes	Yes	Yes
Inning	Yes	Yes	Yes	Yes
Fit statistics				
Observations	7,199,443	7,199,443	7,199,443	7,199,443
R^2	0.05682	0.05682	0.05683	0.05683
Within R^2	0.00781	0.00781	0.00782	0.00782

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Columns (1) and (3) are clustered at the pitcher level and columns (2) and (4) are clustered at the game level. Columns (1) and (3) Speed measure is the pitch's velocity when released by the pitcher in mph. Columns (3) and (4) Speed measure is the natural logarithm of the same variable. # Failures and # Successes count the number of successes and failures since the beginning of the inning. Attempt measures the number of pitches the pitcher has thrown since the beginning of the game. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ Inning Score measures the change in score since the beginning of the inning. P_{type} is a categorical variable indicating the pitch type. Game State is a categorical variable indicating the number of batters out, the number of balls, strikes and whether a player is on base 1, 2 or 3. Player × Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

tion affects the quality of the pitches.[31] In general, pitch quality tends to decrease with higher frustration levels, even though this decrease in quality is marginal. As such, the controllability of the environment does not seem to be the main driver of the emotional reaction, as frustration would increase pitch quality in that case. The following remark summarises the result of this section.

Remark. *Given that pitch velocity does not affect the probability of success in an economically meaningful way, the emotional relief effect will not affect speed choices when frustration increases.*

4.5 Frustration and pitcher's preferences for speed and estimation strategy.

Now that the emotional and rational incentive structure is set, I can describe the pitcher's maximisation problem. The catcher and pitcher first choose the pitch they want to play. I focus on the pitcher's expected utility once this decision is made. Let s_t denote the game's strategic state at time t and let u_{s_t} measure the success utility of a pitch and a state of the game s_t. Let $x(s_t)$ be the pitch speed, with $x^*(s_t) = 0$ being the optimal speed given the pitch type and the strategic environment s_t. Let $\bar{\pi}_{s_t}$ be the probability of success associated with this optimal speed.[32] As the previous section shows, local deviation from the optimal speed does not seem to affect the probability of success meaningfully. As such, in the utility described in (10), local deviations from the optimal speed do not affect the exogenous probability of success $\bar{\pi}_{s_t}$. Finally, I represent the possible appraisal tendency effect of frustration on speed by $\mu(x_t, f_t)$. I assume that $\mu(x_t, 0) = 0$ and $\mu_{xf} > (<)0$ if there are positive or negative appraisal tendencies. The expected utility is:

$$U(x_t, f_t, s_t) = \bar{\pi}_{s_t} \cdot u_{s_t} + \mu(x_t, f_t) - v(f_t) - c(x_t) \tag{10}$$

Equation (10) shows the challenges of this illustration: the success probability and utility are game-state dependent. As such, it is capital to have an efficient control strategy as described in figure (1). Supplemental appendix (F.5) solves a general intertemporal

[31] The quality of the pitches is determined using a machine learning algorithm. I use the physical characteristics of the throw as an ex-ante predictor of success to measure their quality. See supplemental appendix F.2 for more details on this.

[32] One can consider that before normalisation, the optimal speed $x^*_{s_t}$ solves $\bar{\pi}'(x)u_{s_t} = c'(x_t)$.Where $\bar{\pi}(\cdot)$ is some hump-shaped function giving the probability of success for a given speed. The regression considers local deviation from this optimum and $\bar{\pi} = \bar{\pi}(x^*_{s_t})$.

stochastic maximisation problem with instantaneous expected utility (10). The model shows that given that (1) the probability of success is exogenous and (2) frustration increases in exogenous increments, frustration only affects the speed of decisions through the appraisal tendency effect. Any deviation from the optimal speed can directly be attributed to the appraisal tendency effect. Moreover, given that the frustration stock is independent of the pitch velocity, frustration has a static effect. As such, while the pitcher's behaviour can reflect the effect of frustration accumulation, it will not be affected by how he anticipates and manages future frustration. The following remark concludes and clarifies what emotions are affecting behaviour.

Remark. *Frustration affects pitch velocity choices for a given strategic state of the game through the appraisal tendency effect.*

Estimation Strategy

Given this, I estimate the following regression using a multi-way fixed effect estimator:

$$\begin{aligned} Speed_{gpti} &= \beta_0 + \beta_1 F_{gpti} + \beta_2 \Delta Exp._{gpti} \\ &\quad + \beta_3 Game\ State_{gpti} + \beta_4 P_{Type_{gpti}} + \beta_5 Batter_{gpti} + \beta_6 \Delta Score_{gpti} + \beta_7 \Delta InScore_{gpti} \\ &\quad + \beta_8 \#Attempt_{gpti} + \beta_9 \#Failures_{gpti} + \beta_{10} \#Successes_{gpti} \\ &\quad + \beta_{11} \alpha_{gp} + \beta_{12} \alpha_i \end{aligned}$$

Where the subscripts g denote game, p denotes pitcher, t denotes the throw number, and i is the inning, α_{gp} is the $Player \times Game$ fixed effect, α_i is the inning fixed effect.

4.6 Results

4.6.1 Frustration's effect on pitchers' behaviour

Table 3 presents the main result of this section. One additional unit of frustration $\mathbf{F_t}$ (measured in runs) increases the speed of pitches by 0.12 mph (0.19km/h). Depending on the type of pitch thrown, this accounts for an increase in speed between 0.08% to 0.17%.[33] The careful strategy to control for cofounders also seems to work with an adjusted R^2 of 0.912.[34].

[33] See table F.1.2, in the supplemental appendix, for more detailed results

[34] Admittedly, the frustration measure has a negligible impact on the adjusted R^2 of the remaining variance to explain. This is unsurprising and similar to the explanatory power in causal identification

To better understand the magnitude of the effect, let us compare it to the impact of fatigue, as defined in Table 1. The effect of one standard deviation increase in frustration (0.27 runs) accounts for 45.72% of the magnitude of one standard deviation increase in the fatigue gained during an inning by a pitcher (5.16 throws). Suppose one considers the standard deviation of frustration for positive frustration values only. In that case, the effect rises to 64.70 % of one standard deviation increase in fatigue[35]. As such, frustration seems to have a sizable impact on pitcher's behaviour.

Emotional Relief

One crucial building block of the model is that success brings emotional relief, in that frustration's effect disappears after success. Notice the frustration coefficient is positive and significant while controlling for Δ Exp.. In other words, the impact of frustration cannot be captured by a net effect of an elation/frustration function that Δ Exp. (the net difference in run expectancy since the beginning of the inning) should capture. One can interpret this as evidence of an asymmetric drop in F_t after a success. There must be a relief and accumulation effect, as in F_t's construction. Whether frustration really goes to 0 after success is an open question, however if anything this would lead to an underestimation of frustration's effect on speed.

Temporal Effects

Finally, the central insight from this paper is that emotions are inter-temporal processes: frustration accumulates and decays with time. Table 1 analysis shows that F_t, affects pitchers' behaviour. However, it does not explore the temporal effect of frustration accumulation. F_t can increase by the same amount over one or several failures. If frustration has a temporal effect, then the marginal frustration Δf_t gained several failures ago (as defined in equation (8)) should still affect pitches' speed in case of consecutive failures.

As such, I first construct a set of dummy variables indicating whether there were more than 1,2 and up to 9+ consecutive failures. I then interact the dummy, indicating one or more failures with frustration gained last period, two or more failures with the frustration gained two periods ago and so on. Generally, for an observation at time t, the interactions are:

techniques after differencing away the trends.

[35]This is a sensible strategy as the frustration stock equals 0 in 80% of the sample

Table 3: Effect of Frustration on Velocity

Dependent Variable:	Velocity	
Model:	(1)	(2)
Variables		
$\mathbf{F_t}$	0.1221***	0.1221***
	(0.0048)	(0.0034)
# Failures	0.0189***	0.0189***
	(0.0014)	(0.0008)
# Successes	0.0345***	0.0345***
	(0.0016)	(0.0007)
Attempt	-0.0143***	-0.0143***
	(0.0012)	(0.0003)
Δ Exp.	0.0270***	0.0270***
	(0.0071)	(0.0062)
Δ Score	0.0007	0.0007
	(0.0021)	(0.0012)
Δ Inning Score	-0.0887***	-0.0887***
	(0.0075)	(0.0060)
Fixed-effects		
P_{type}	Yes	Yes
Game State	Yes	Yes
Player $\times$ Game	Yes	Yes
Batter	Yes	Yes
Inning	Yes	Yes
Fit statistics		
Observations	7,199,443	7,199,443
R^2	0.91726	0.91726
Within R^2	0.00235	0.00235

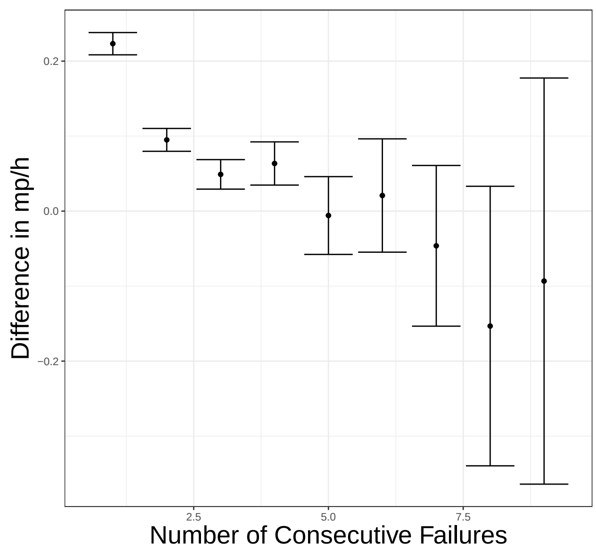
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Column (1) is clustered at the pitcher level, and column (2) at the game level. $\mathbf{F_t}$ measures the change in the other team's expected score during the current and (possibly empty) sequence of consecutive failure. # Failures and # Successes count the number of successes and failures since the beginning of the inning. *Attempt* measures the number of pitches the pitcher has thrown since the start of the inning. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ InScore measures the change in score since the beginning of the inning. P_{type} is a categorical variable indicating the pitch type. Game State is a categorical variable indicating the number of batters out, the number of balls, strikes and whether a player is on base 1, 2 or 3. Player $\times$ Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

$$\mathbb{1}[\#Consecutive\,Failures \geq n] \times \Delta f_{t-n}$$

These interactions allow the n^{th} frustration lag to have an effect only if there were at least n^{th} consecutive failures, like in the model. The $n > 1$ coefficients should be significant if frustration accumulates over time. Moreover, remember that the measured frustration law of motion (9) does not include a decay rate. As such, if frustration decays with time, the magnitude of the interactions' coefficients should decrease with the number of lags. Intuitively, if it decays, the frustration gained five pitches ago will have a lower impact on speed than the frustration stemming from the last failure.

Figure 3: Temporal Effect of Frustration



Each dot on the graph represent the interaction term $\mathbb{1}[\#Consecutive\,Failures \geq n] \times \Delta f_{t-n}$ for $1 \leq n \leq 8$. The 9^th dot represents the interaction between lags of Δf greater or equal to 9 and the dummy indicating more than nine consecutive failures. The confidence interval displayed on the graph are associated with 5% confidence level. Clusters are at the player level.

Figure 3 represents the value of the coefficients of the interactions and their confidence intervals (95%) for an OLS with these interactions, the dummy indicating the number of cumulative failures and the set of control presented in Table 2. Frustration seems to have a temporal effect, with frustration from up to four failures ago still significantly impacting speed (at a 5% significance level). Second, frustration also seems to decay as the coefficient value gradually decreases with non-significant values if failures occurred more than three throws ago, indicating a substantial decay rate of frustration over time.

Empirical conclusion:

Overall, the analysis points to appraisal tendencies as the main culprit behind the emotional reaction. Frustrated pitchers seem to throw faster because they have a higher marginal utility to do so. In supplemental appendix F.3, I carry out the same analysis at the individual level and take a closer look at frustration's effect on the pitches' quality. A higher effect of frustration on speed is associated with a more negative impact on quality. This rules out any rationale for saying that frustration motivates pitchers to reach a better optimum and strengthens the conclusion that appraisal tendencies are the source of the changes. In the same supplemental appendix, I show that frustration's effect on quality also correlates with lower career-level performance statistics. This indicates that frustration accumulation is more than an interesting behavioural phenomenon and could have labour market implications by affecting pitchers' productivity.

5 Conclusion and Discussion

This paper establishes a framework to study the inter-temporal effects of emotions on investment. I find that the impact of frustration on investment depends mainly on the relative force of the negative emotional cost, appraisal tendencies and the prospective emotional relief effect. More importantly, negative emotions do not always trigger negative consequences in terms of investment, and positive appraisal tendencies stemming from them are not necessary nor sufficient to have a positive effect of frustration on motivation. While in line with the psychological literature, the framework shows that one must be cautious when inferring behavioural implications from negative emotions.

Overall, the empirical investigation seems to confirm that pitcher's behaviour is influenced by frustration and that frustration accumulates and decays. Frustration appears to increase pitches' speed at the average and individual levels. For pitchers who are the most affected, frustration seems to affect pitcher's career-level performance and productivity. This highlights the relevance of studying the dynamics of emotions in economics.

The framework presents two assumptions that require some discussion. First, the decision-maker is forward-looking. This assumption is not innocuous but standard in the literature (Loomes and Sugden, 1982; Bell, 1982, 1985; Loomes and Sugden, 1986; Caplin and Leahy, 2001). Yet, while individuals anticipate their emotions reasonably well, they are also prone to systematic mistakes (Loewenstein, 1999). Nonetheless, the mechanisms and trade-offs presented here are surprisingly intuitive and show that some "forward-lookingness" must be present to get intuitive behaviours. For example, if an agent cannot predict that her frustration will disappear after a success, investing more effort to solve the problem would not make sense.

Second, the decision-maker is time-consistent. A decision-maker might indeed decide to use some commitment device to limit the action-set of her future frustrated self. However, while intuitively appealing, the approach leads to counterintuitive conclusions. In particular, a self with $f = 0$ will always ignore her action's effect on future frustration accumulation when setting her investment plan. Indeed, her only criteria for evaluating future investment will be her current preference relation, which does not feature any frustration. Similarly, a self with $f > 0$ will also neglect the positive impact of getting emotional relief on utility. As such, while interesting, it is not straightforward to bring and conceptualise time-inconsistent behaviour stemming from emotional dynamics. I see this work as a helpful benchmark and stepping stone for future work in that area.

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Supplemental Appendix for Negative Emotion Accumulation and Personal Motivation

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A Proofs and solutions to the main model

A.1 Solution to the main model

In this section, I solve the general version of the model. To make the problem more tractable, let me introduce the following state variable: $\Phi_t = \phi(x_t, t) = \int_0^t \gamma x_s ds + rt$, where r is the time-invariant discount rate, which might include a Poisson law fixed arrival rate π. In section 3.1, I have $\gamma = 0$ and $r = \rho + \pi$. In section 3.2, $\gamma > 0$ and $r = \rho$. I can then rewrite the maximisation problem in the following way:

$$
\begin{aligned}
V &= \max_{x_t} \int_0^{\infty} e^{-\Phi_t} U(x_t, f_t) dt \\
\dot{f}_t &= x_t - \delta f_t \\
\dot{\Phi}_t &= \gamma \cdot x_t + r \\
&f_{t_0} \text{ given} \\
&\Phi_0 = 0
\end{aligned}
$$

The Hamiltonian of this problem is:

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$$\tilde{H}(x_t, f_t, \tilde{\lambda}_t, \tilde{\mu}_t) = e^{-\Phi_t}U(x_t, f_t) - \tilde{\lambda}_t(x_t - \delta f_t) - \tilde{\mu}_t(\gamma \cdot x_t + r)$$

And the first order conditions of the maximisation problem are:

$$\begin{aligned}
&\tilde{H}_x(\cdot) = e^{-\Phi_t}U_x(x_t, f_t) - \tilde{\lambda}_t - \tilde{\mu}_t\gamma = 0\\
&\tilde{H}_f(\cdot) = e^{-\Phi_t}U_f(x_t, f_t) + \delta\tilde{\lambda}_t = \dot{\tilde{\lambda}}_t\\
&\tilde{H}_\Phi(\cdot) = -e^{-\Phi_t}U(x_t, f_t) = \dot{\tilde{\mu}}_t\\
&\lim_{t\to\infty} \tilde{H}(\cdot) = 0
\end{aligned}$$

Where the last equation is a general transversality condition suited for endogenously discounted inter-temporal maximisation problem see (Michel, 1982; Barro and Sala-i Martin, 2004). Next, notice that:

$$\frac{d\tilde{H}(\cdot)}{dt} = \dot{x}_t\tilde{H}_{x_t}(\cdot) + \dot{f}_t\tilde{H}_{f_t}(\cdot) + \dot{\tilde{\Phi}}_t\tilde{H}_{\Phi_t}(\cdot) - \dot{\tilde{\mu}}_t\dot{\Phi}_t - \dot{\lambda}_t\dot{f}_t = 0$$

Together with the transversality condition, this means that on the optimal path, the maximised Hamiltonian $\tilde{H}^*(\cdot)$ is always equal to 0. As such, I can rewrite:

$$\tilde{\mu}_t = \frac{e^{-\Phi_t}U(x_t, f_t) - \tilde{\lambda}_t(x_t - \delta f_t)}{\gamma \cdot x_t + r}$$

Let me introduce the following variables : $\tilde{\mu}_t = e^{-\Phi_t}\mu_t$ and $\tilde{\lambda}_t = e^{-\Phi_t}\lambda_t$.

The first order conditions become:

$$U_x(x_t, f_t) - \lambda_t - \mu_t\gamma = 0 \tag{A.1}$$

$$U_f(x_t, f_t) + (\delta + r + \gamma \cdot x_t)\lambda_t = \dot{\lambda}_t \tag{A.2}$$

$$-U(x_t, f_t) + (\gamma \cdot x_t + r)\mu_t = \dot{\mu}_t \tag{A.3}$$

$$\lim_{t\to\infty} \tilde{H}(\cdot) = 0 \tag{A.4}$$

The transversality condition (A.4) then implies that:

$$\mu_t = \frac{U(x_t, f_t) - \lambda_t(x_t - \delta f_t)}{\gamma \cdot x_t + r} \tag{A.5}$$

To make the following computations more tractable, let me drop the time index and the argument of the expected utility $U(\cdot)$ and of its derivatives. Inserting (A.5) in the first order condition (A.1) to solve for μ and λ yields:

$$\begin{aligned}
& U_x = \lambda + \gamma \frac{U - \lambda(x - \delta f)}{\gamma \cdot x + r} \\
\iff & (\gamma \cdot x + r) U_x = (\gamma \cdot x + r)\lambda + \gamma \left(U - \lambda(x - \delta f)\right) \\
\iff & (\gamma \cdot x + r) U_x - \gamma U = \left(\gamma \cdot x + r - \gamma(x - \delta f)\right) \lambda \\
\iff & \lambda = \frac{(\gamma \cdot x + r) U_x - \gamma U}{r + \gamma \delta f}
\end{aligned} \tag{A.6}$$

If I insert this result back in (A.5):

$$\begin{aligned}
\mu &= \frac{U - \lambda(x - \delta f)}{\gamma \cdot x + r} \\
&= \frac{U}{\gamma \cdot x + r} - \frac{(x - \delta f)}{\gamma \cdot x + r}\lambda \\
&= \frac{U(\gamma \cdot x + r) - (x - \delta f)(\gamma \cdot x + r) U_x}{(r + \gamma \delta f)(\gamma \cdot x + r)} \\
&= \frac{U - (x - \delta f) U_x}{(r + \gamma \delta f)}
\end{aligned}$$

As such, I can now get the re-express the co-state law of motion μ described in (A.3) as:

$$\begin{aligned}
\dot{\mu} &= -U + (\gamma \cdot x + r) \frac{U - (x - \delta f) U_x}{r + \gamma \delta f} \\
&= \frac{-U(r + \gamma \delta f) + (\gamma \cdot x + r)(U - U_x \dot{f})}{r + \gamma \delta f} \\
&= -\frac{\gamma U (x - \delta f) - (\gamma \cdot x + r) \left(U_x \dot{f}\right)}{r + \gamma \delta f} \\
\dot{\mu} &= -\lambda \dot{f}
\end{aligned} \tag{A.7}$$

Let me now differentiate the first order condition A.1 with regards to time to get the Euler equation :

$$U_{xx}\dot{x} - \dot{\lambda} - \gamma\dot{\mu} + \dot{f}U_{xf} = 0$$
$$\Longleftrightarrow \dot{x} = \frac{1}{U_{xx}}\left(\dot{\lambda} + \gamma\dot{\mu} - U_{xf}\dot{f}\right) \tag{EE}$$

The Euler equation (EE), together with the frustration law of motion (2) form the canonical system of the inter-temporal maximisation problem. Next, let me linearise the system around the steady state to study its dynamics. The linearised system is given by:

$$\begin{pmatrix} \dot{\hat{x}}_t \\ \dot{\hat{f}}_t \end{pmatrix} = J^* \begin{pmatrix} \hat{x}_t \\ \hat{f}_t \end{pmatrix}.$$

Where the variables denoted by hats are the rescaled investment and frustration, such that steady state value are normalized to 0. J^* is the Jacobian of the canonical system evaluated at the steady-state (x^*, f^*):

$$J^* = \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f)}{\partial x} & \frac{\partial \dot{x}(x,f)}{\partial f} \\ \frac{\partial \dot{f}(x,f)}{\partial x} & \frac{\partial \dot{f}(x,f)}{\partial f} \end{pmatrix}\right|_{(x,f)=(x^*,f^*)}$$

To get the expression of J^*, I need to compute the value of the derivative of λ in (A.6) with regard to f and x at the steady state:

$$\left.\frac{\partial \lambda}{\partial f}\right|_{(x^*,f^*)} = \frac{\partial}{\partial f}\left.\left(\frac{(\gamma \cdot x + r)U_x - \gamma U}{r + \gamma\delta f}\right)\right|_{(x^*,f^*)}$$
$$= U_{xf}^* - \frac{\gamma\left(U_f^* + \delta\lambda^*\right)}{(r + \gamma \cdot x^*)} \tag{A.8}$$
$$\left.\frac{\partial \lambda}{\partial x}\right|_{(x^*,f^*)} = U_{xx}^* \tag{A.9}$$

Next using (A.8) and (A.9), I can differentiate the co-state law of motion (A.2) with regards to x and f

$$\left.\frac{\partial\dot{\lambda}}{\partial f}\right|_{(x^*,f^*)} = \frac{\partial}{\partial f}\left(U_f + (\delta + r + \gamma \cdot x^*)\lambda\right)$$
$$= U^*_{ff} + (\delta + r + \gamma \cdot x^*)\frac{\partial\lambda}{\partial f}$$
$$= U^*_{ff} + (r + \delta + \gamma \cdot x^*)\left(U^*_{xf} - \frac{\gamma\left(U^*_f + \delta\lambda^*\right)}{(r + \gamma \cdot x^*)}\right)$$
$$\left.\frac{\partial\dot{\lambda}}{\partial x}\right|_{(x^*,f^*)} = \frac{\partial}{\partial x}\left(U_f + (\delta + r + \gamma \cdot x)\lambda\right)$$
$$= U^*_{xf} + \gamma\lambda + (r + \delta + \gamma \cdot x^*)U^*_{xx}$$

I can finally compute the elements of Jacobian matrix J^* using equations (A.6)-(A.9):

$$\left.\frac{\partial\dot{x}}{\partial f}\right|_{(x^*,f^*)} = \frac{\left(U_{xx}\frac{\partial}{\partial f}\left(\dot{\lambda} + \gamma\dot{\mu} - U_{xf}\dot{f}\right) - U_{xxf} \cdot (0)\right)}{U^2_{xx}}$$
$$= \frac{1}{U^*_{xx}}\left(\frac{\partial}{\partial f}\left(\dot{\lambda} + \gamma\dot{\mu} - U_{xf}\dot{f}\right)\right)$$
$$= \frac{1}{U^*_{xx}}\left(U^*_{ff} + (\delta + r + \gamma \cdot x^*)\frac{\partial\lambda}{\partial f} + \gamma\delta\lambda^* + \delta U^*_{xf}\right)$$
$$= \frac{1}{U^*_{xx}}\left(U^*_{ff} + (r + \delta + \gamma \cdot x^*)\left(U^*_{xf} - \frac{\gamma\left(U^*_f + \delta\lambda^*\right)}{(r + \gamma \cdot x^*)}\right) + \gamma\delta\lambda^* + \delta U^*_{xf}\right)$$
$$= \frac{1}{U^*_{xx}}\left(U^*_{ff} + (r + \gamma \cdot x^* + 2\delta)\left(U^*_{xf} - \frac{\gamma U^*_f}{r + \gamma \cdot x^* + \delta}\right)\right)$$

Where I insert λ^*'s steady-state value determined by (A.2) in the two last lines. Similarly, I take the derivative of the Euler equation at the steady state with regard to investment.

$$
\begin{aligned}
\left.\frac{\partial \dot{x}}{\partial x}\right|_{(x^*,f^*)} &= \frac{\left(U_{xx}\frac{\partial}{\partial x}\left(\dot{\lambda}+\gamma\dot{\mu}+U_{xf}\dot{f}\right)-U_{xxx}\cdot(0)\right)}{\left(U_{xx}\right)^2}\\
&= \frac{U^*_{xx}\left(U^*_{xf}+\gamma\lambda+(r+\delta+\gamma\cdot x^*)U^*_{xx}-\gamma\lambda-U^*_{xf}\right)}{{U^*_{xx}}^2}\\
&= r+\delta+\gamma\cdot x^*
\end{aligned}
$$

Let $\Omega^*_f = \left.\frac{\partial \dot{x}(x,f)}{\partial f}\right|_{(x^*,f^*)}$. Putting everything together the Jacobian of the system evaluated at the steady-state is:

$$
J^* = \begin{pmatrix} r+\delta+\gamma\cdot x^* & \Omega^*_f \\ 1 & -\delta \end{pmatrix}
$$

The eigenvalue of the Jacobian are given by:

$$
\begin{aligned}
\mu_{1,2} &= \frac{tr(J^*)\pm\sqrt{tr(J^*)^2-4\nabla(J^*)}}{2}\\
&= \frac{(\gamma\cdot x^*+r)\pm\sqrt{(\gamma\cdot x^*+r+2\delta)^2+4\Omega^*_f}}{2}. \qquad \text{(A.10)}
\end{aligned}
$$

Where $\nabla(J^*)$ is the determinant of the Jacobian matrix J evaluated at the steady-state.

A.2 Proofs

A.2.1 Proposition 1

Let me set $\gamma = 0$ and $r = \pi + \rho$ using (A.10), I get:

$$
\mu_{1,2} = \frac{r\pm\sqrt{(r+2\delta)^2+4\Omega^*_f}}{2}.
$$

In this special case, I have $\Omega^*_f = \Omega^*_A = \frac{1}{U_{xx}}\left(U_{ff}+(r+2\delta)U_{xf}\right)$

For (1), consider the optimal path of the frustration associated with the smallest eigenvalue μ_1.

$$\hat{f}_t = ke^{\mu_1 t} \tag{A.11}$$

$$\dot{\hat{f}}_t = k\mu_1 e^{\mu_1 t} \tag{A.12}$$

Where $k < 0$, because $f_{t_0} < f^*$. Using the frustration law of motion to express x_t and substituting (A.11) and (A.12) yields

$$\hat{x}_t = (\delta + \mu_1)k \cdot \hat{f}_t$$

$k \cdot \hat{f}_t > 0$, as such, $\hat{x}_t$, will be increasing in $\hat{f}_t$ if

$$(\delta + \mu_1) > 0$$
$$\iff \frac{r + 2\delta - \sqrt{(r + 2\delta)^2 + 4\Omega_A^*}}{2} > 0 \iff \Omega_A^* < 0$$

And similarly $\hat{x}_t$, will be decreasing in $\hat{f}_t$ if $\Omega_A^* > 0$. For (2), saddle path stability requires one positive and one negative eigenvalue. The highest eigenvalue will always be positive. On the other hand, the lowest will be negative if and only if:

$$r < \sqrt{(r + 2\delta)^2 + 4\Omega_A^*}$$
$$\iff r^2 - r^2 - 4\delta(r + \delta) - 4\Omega_A^* < 0$$
$$\iff -\delta(r + \delta) < \Omega_A^*$$

A.2.2 Proposition 2

It is a special case of the general model. It follows the same logic as the proof of Proposition 1, with $\gamma > 0$, $r = \rho$.

B Frustration accumulation with exogenous increments

This section explores the implication of frustration accumulation when frustration increases by an exogenous amount $a > 0$ when the decision maker fails. As such,the maximisation problem is identical to the main formulation (G.O) presented in section 3 in the main text with an adapted law of motion:

$$\dot{f}_t = a - \delta f_t. \tag{B.1}$$

With $a > 0$. The main difference is that the frustration cost of failing is independent of the investment level. The Hamiltonian of the maximisation problem is:

$$\tilde{H}(x_t, f_t, \tilde{\lambda}_t, \tilde{\mu}_t) = e^{-\Phi_t} U(x_t, f_t) - \tilde{\lambda}_t(a - \delta f_t) - \tilde{\mu}_t(\gamma \cdot x_t + r)$$

Using the same change of variable and reasoning as in section A.1, with $\tilde{\mu}_t = e^{-\Phi_t}\mu_t$ and $\tilde{\lambda}_t = e^{-\Phi_t}\lambda_t$, I get that the same first order conditions (A.2)-(A.4) and (A.1) becomes $U_x(x_t, f_t) = \gamma\mu_t$. Differentiating the latter, I get that:

$$\begin{aligned}
&\dot{x}_t = \frac{1}{U_{xx}(x_t, f_t)} \left(\gamma\dot{\mu}_t - U_{xf}(x_t, f_t)\right) \\
\iff &\dot{x}_t = \frac{1}{U_{xx}(x_t, f_t)} \left(\gamma\left(-U(x_t, f_t) + (\gamma \cdot x_t + r)\mu_t\right) - U_{xf}(x_t, f_t)\dot{f}_t\right) \\
\iff &\dot{x}_t = \frac{1}{U_{xx}(x_t, f_t)} \left(\gamma\left(-U(x_t, f_t) + (\gamma \cdot x_t + r)\frac{U_x(x_t, f_t)}{\gamma}\right) - U_{xf}(x_t, f_t)\dot{f}_t\right) \qquad \text{(B.2)}
\end{aligned}$$

The Euler equation (B.2), together with the frustration law of motion (B.1) form the canonical system of the inter-temporal maximisation problem. Next, let me linearise the system around the steady state to study its dynamics. These are given by

$$\begin{pmatrix} \dot{\hat{x}}_t \\ \dot{\hat{f}}_t \end{pmatrix} = J^* \begin{pmatrix} \hat{x}_t \\ \hat{f}_t \end{pmatrix}$$

Where the variables denoted by hats are the rescaled investment and frustration, such that steady state value are normalized to 0. J^* is the Jacobian of the canonical system evaluated at the steady-state (x^*, f^*):

$$\begin{aligned}
J^* &= \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f)}{\partial x} & \frac{\partial \dot{x}(x,f)}{\partial f} \\ \frac{\partial \dot{f}(x,f)}{\partial x} & \frac{\partial \dot{f}(x,f)}{\partial f} \end{pmatrix}\right|_{(x,f)=(x^*,f^*)} \\
&= \begin{pmatrix} \gamma x + r & \Omega_f^* \\ 0 & -\delta \end{pmatrix}
\end{aligned}$$

Where

$$\Omega_f^* = \frac{1}{U_{xx}^*}\left((r+\gamma x+\delta)U_{xf} - \frac{\gamma U_f^*}{r+\gamma x+\delta}\right)$$

The effect of an increase in frustration on the investment level (around the steady state) has a very familiar form. In particular, it is influenced by the two same behavioural forces as in the general case, the prospective emotional relief effect and the appraisal tendency effect. On the other hand, the emotional cost is absent. This means that when frustration increases in discrete and exogenous steps when the decision maker fails, she will not try to reduce her investment level to protect herself from future frustration costs. The intuition behind this result is simple. Whether she invested a lot or a little doing the last failed attempt will not affect her frustration cost after that. As such, her investment level is independent of the frustration marginal cost level given by the local curvature of U_{ff}.

C Rational Addiction without habit formation

In this section, I focus on the possible addictive tendencies frustration can create with regard to investment. Addiction and emotions have a strong link, which has been studied qualitatively in economics (Loewenstein, 1999; Elster, 1999). This is the first contribution to formally establishing this link. It would be straightforward to show that one can get a rational addiction model with positive appraisal tendencies as the mechanics are the same as the one studied in the original formulation (Becker and Murphy, 1988). As such, I focus on cases with no appraisal tendencies, with $u_{xf} = 0$ and where investment increases the probability of success, as in section 3.2. In this case, the dynamics of the model will stem from a mechanism that is absent in rational addiction models: the prospective emotional relief effect. I can write the inter-temporal decision problem as:

$$\begin{aligned} V &= \max_{x_t \in [0,I]} \int_0^\infty e^{-\phi(x_t,t)} U(x_t, f_t) dt \\ \dot{f}_t &= x_t - \delta f_t \\ &f_0 \text{ given} \end{aligned} \tag{G.O}$$

Where $\phi(x_t, t) = \int_0^t \gamma \cdot x_s ds + \rho t$ is the effective discount rate and $\rho > 0$ is the subjective discount rate. We want the maximisation problem to be concave in x for any value of f. Endogenously discounted inter-temporal maximisation problems are solvable but become complicated very quickly. As such, I will focus on a relatively simple functional form.

$$U(x_t, f_t) = -W - \beta\frac{f_t^2}{2} - \frac{c}{2}x_t^2 \tag{C.1}$$

Equation (C.1) describes the preferences of an agent suffering a utility loss $-W$, $W > 0$, that she could eliminate by achieving success. As such, the agent is willing to invest in increasing the odds of eliminating this utility cost. On the other hand, the investment and frustration costs remain the same as in the main text. The next lemma shows that our maximisation problem is well-defined as long as $W \geq \frac{c}{\gamma^2}$.

Lemma C.1. *The integrand in* (G.O) *is concave in x and f (separately) if $W \geq \frac{c}{\gamma^2}$.*

In what follows I assume that $W \geq \frac{c}{\gamma^2}$. Let me introduce a new state variable $\Phi_t = \int_0^t \gamma \cdot x_s ds + \rho t$ and rewrite the maximisation problem as:

$$\begin{aligned}
V &= \max_{x_t \in [0,I]} \int_0^\infty e^{-\Phi_t} U(x_t, f_t) dt \\
\dot{f}_t &= x_t - \delta f_t \\
\dot{\Phi}_t &= \gamma x_t + \rho \\
& f_0 \text{ given}
\end{aligned}$$

The current value Hamiltonian of the maximisation problem is:

$$H(t, f_t, x_t, \tilde{\lambda}_t, \tilde{\mu}_t) = e^{-\Phi_t}\left(-W - \alpha\frac{f_t^2}{2} - \frac{c}{2}x_t^2\right) - \tilde{\lambda}_t(x_t - \delta f_t) - \tilde{\mu}_t(\gamma x_t + \rho)$$

Moreover, the Lagrangian is:

$$L(t, f_t, x_t, \lambda_t, \psi_t) = H(t, f_t, x_t, \lambda_t) + \psi_t^1(I - x) - \psi_t^2 x_t.$$

Where ψ_t^1 and ψ_t^2 are the Lagrangian multiplier associated with the Kuhn and Tucker conditions for optimality. The first-order conditions for a point interior to the domain are:

$$
\begin{cases} H_x = 0 \\ \dot{\tilde{\lambda}}_t = H_f \\ \dot{\tilde{\mu}}_t = H_\Phi \\ \lim_{t\to\infty} \tilde{H}(\cdot) = 0 \end{cases} \iff \begin{cases} -e^{-\Phi_t} \cdot c \cdot x_t = \tilde{\lambda}_t + \gamma \tilde{\mu}_t \\ \dot{\tilde{\lambda}}_t = -e^{-\Phi_t} \beta f_t - \delta \tilde{\lambda}_t \\ \dot{\tilde{\mu}}_t = -e^{-\Phi_t} \left(-W - \beta \frac{f_t^2}{2} - \frac{c}{2} x_t^2 \right) \\ \lim_{t\to\infty} \tilde{H}(\cdot) = 0 \end{cases} \tag{FOC}
$$

Using the same reasoning as in section A.1, we can rewrite our system as:

$$
\begin{cases} -c \cdot x_t - \gamma \mu_t = \lambda_t \\ \dot{\lambda}_t = -\beta f_t + (\rho + \gamma \cdot x + \delta) \lambda_t \\ \dot{\mu}_t = -W - \beta \frac{f_t^2}{2} - \frac{c}{2} x_t^2 + (\rho + \gamma \cdot x) \mu_t \\ \mu_t = \dfrac{-W - \beta \frac{f_t^2}{2} - \frac{c}{2} x_t^2 - \lambda_t (x_t - \delta f_t)}{\gamma \cdot x_t + \rho} \end{cases} \tag{FOC2}
$$

Solving for μ_t and λ_t using the first and last equations of (FOC2):

$$
\begin{cases} \lambda_t = \dfrac{\gamma (W + \beta \frac{f_t^2}{2} + \frac{c}{2} x_t^2) - c \cdot x_t (\gamma x_t + \rho)}{\rho + \gamma \delta f_t} \\ \mu_t = \dfrac{-W - \beta \frac{f_t^2}{2} - \frac{c}{2} x_t^2 + c \cdot x_t (x_t - \delta f)}{(\rho + \gamma \delta f_t)} \end{cases} \tag{CS}
$$

Using the expressions for the co-states (CS), I can rewrite the co-states law of motions as:

$$
\begin{cases} \dot{\lambda}_t = -\beta f_t + (\rho + \gamma \cdot x_t + \delta) \dfrac{\gamma (W + \beta \frac{f_t^2}{2} + \frac{c}{2} x_t^2) - c \cdot x (\gamma x + \rho)}{\rho + \gamma \delta f} \\ \dot{\mu}_t = -\dfrac{\gamma (W + \beta \frac{f_t^2}{2} + \frac{c}{2} x_t^2) - c \cdot x_t (\gamma x_t + \rho)}{\rho + \gamma \delta f_t} \cdot (x - \delta f_t) = -\lambda \dot{f}_t \end{cases} \tag{CSLM}
$$

Differentiating the first equation in (FOC2), I get the canonical system of our intertemporal decision-making problem:

$$
\dot{x}_t = -\frac{1}{c} \left(-\beta f_t + (\rho + \delta + \gamma \cdot x_t - \gamma \dot{f}_t) \lambda_t \right) \tag{EE}
$$

$$
\dot{f}_t = x_t - \delta f_t \tag{F}
$$

Where λ_t is defined in CS. To find the steady state of this system, we must equalise (EE) and (F) to 0 simultaneously. To get a somewhat simpler expression, I assume that $\rho = \delta$[1].

[1]The assumption is not needed but simplifies equation (C.2) by having an unambiguously negative

I get that $f_t = \frac{x_t}{\delta}$. On the other hand, the Euler equation becomes:

$$\frac{\gamma^2}{2}\left(\frac{\beta}{\delta^2} - c\right) x_t^3 + -2\gamma\delta c x_t^2 + \left(\gamma^2 W - \beta - 2\delta^2 c\right) x + 2\delta\gamma \cdot W = 0 \tag{C.2}$$

It is possible to characterise the exact solutions of the system, but they are of little analytic value, given the complexity of their expressions. It is, however, possible to have a good intuition of when and why multiple equilibria might occur. Observe that the constant of the polynomial is positive; as such, to have two positive steady states, the coefficient of the cubic term must be positive, with $\frac{\beta}{\delta^2} > c$. On the other hand, if $\frac{\beta}{\delta} < c$, the steady-state will always be stable and unique. As such, the frustration sensitivity parameter β will naturally drive the addictive tendencies and the system's instability. The higher the frustration, the higher the incentive to eliminate its cost. The following proposition gives the conditions for a steady state to be stable. In particular, it shows that in the case $\frac{\beta}{\delta^2} > c$, a steady state characterised by an investment level that is above a certain threshold will be unstable.

Proposition C.1. *Let $\frac{\beta}{\delta^2} > c$, if they exist, the interior steady-state(s) (x^*, f^*) of the system will be stable if and only if $x^* < \hat{x}$, for some $\hat{x} > 0$ If $\frac{\beta}{\delta^2} \leq c$, the steady-sate (x^*, f^*) is always stable.*

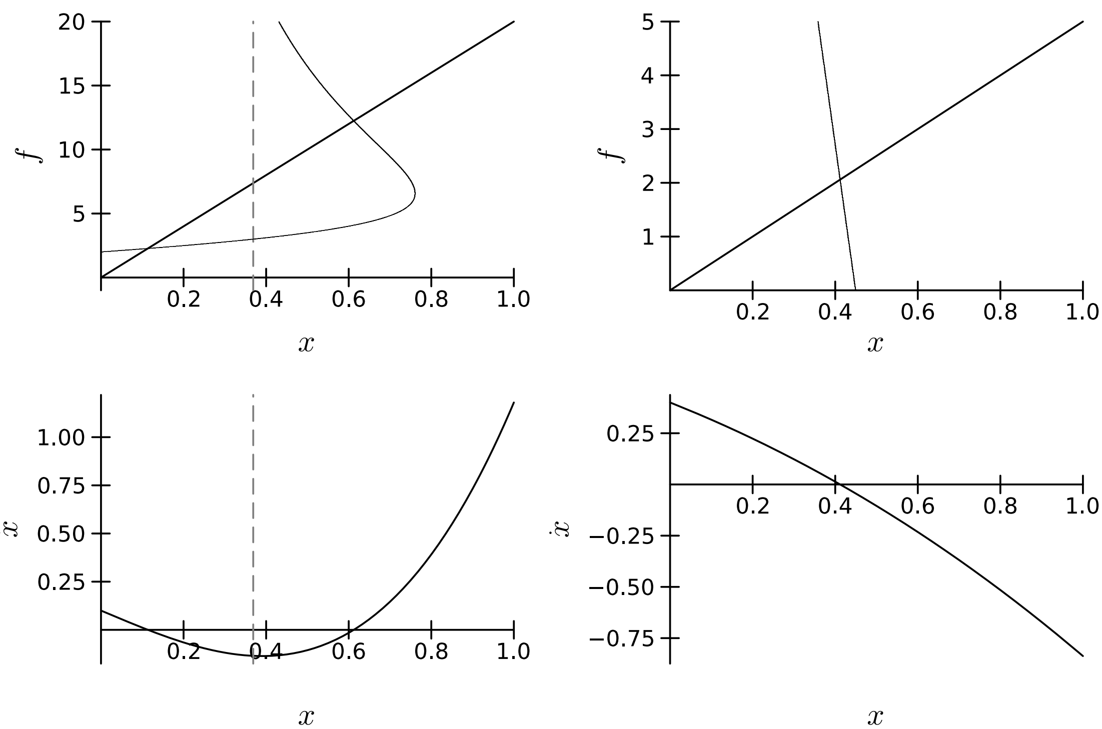
Figure C.0.1 describes two possible cases. The left-hand side of the panel is the typical rational addiction case, with multiple steady states or unstable steady states. The top left panel describes a situation where the low steady-state has an x value below $\hat{x}$ and is stable. The second steady-state is unstable on the right of $\hat{x}$. The right panels show that it is also possible to have a well-behaved maximisation problem where the decision-maker has a single (and stable) steady state.

C.1 Lemma C.1

We need to check under which condition the second order derivative, with regards to x_t, of $e^{-\phi(x_t,t)}\left(-W - \beta\frac{f_t^2}{2} - \frac{c}{2}x_t^2\right)$ is negative.

coefficient for the quadratic coefficient.

Figure C.0.1: Investment dynamics simulations



The upper panels graph (EE) and (F).The light grey line represents (F), and the dark line represents (EE). The dotted line always represents the $\hat{x}$ value. The bottom panels graph the left-hand side of equation(C.2). The left panels present the investment dynamics for an individual with $\gamma = 0.1$, $\beta = 1$, $\delta = 0.05$, $W = 10$ and $c = 1$. The right panels represent a decision-maker with $\gamma = 0.1$, $\beta = 0.1$, $\delta = 0.2$, $W = 10$ and $c = 10$.

$$\frac{\partial e^{-\phi(x_t,t)}U(x,f)}{\partial x} = e^{-\phi(x_t,t)}\left(-c\cdot x_t - \gamma\left(-W - \beta\frac{f_t^2}{2} - \frac{c}{2}x_t^2\right)\right)$$
$$\frac{\partial^2 e^{-\phi(x_t,t)}U(x,f)}{\partial x^2}(x,f) = e^{-\phi(x_t,t)}\left(2\gamma c\cdot x_t - c + \gamma^2\left(-W - \beta\frac{f_t^2}{2} - \frac{c}{2}x_t^2\right)\right) \tag{C.3}$$

Clearly, (C.3) is more likely to become positive if $f = 0$. Let me check if (C.3) is indeed negative even when it maximised. To do this I only need to look at the sign of the expression multiplying $e^{-\phi(x_t,t)}$. Define $G(x) = 2\gamma c\cdot x_t - c + \gamma^2\left(-W - \beta\frac{f_t^2}{2} - \frac{c}{2}x_t^2\right)$. Clearly it is a concave function and $G(0) < 0$. Now let me maximise $G(x)$, the first order conditions gives $x = \frac{2}{\gamma}$. As such, $G(\frac{1}{\gamma}) = c - \gamma^2\cdot W$. As such, the $e^{-\phi(x,t)}U(x,f)$ is always concave in x. On the other hand, $e^{-\phi(x,t)}U(x,f)$ is clearly concave in f.

C.2 Proposition C.1

We must compute the Jacobian matrix values at the possible steady states. Let J^* be this Jacobian of the canonical system evaluated at the steady-state (x^*, f^*):

$$J^* = \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f)}{\partial x} & \frac{\partial \dot{x}(x,f)}{\partial f} \\ \frac{\partial \dot{f}(x,f)}{\partial x} & \frac{\partial \dot{f}(x,f)}{\partial f} \end{pmatrix}\right|_{(x,f)=(x^*,f^*)}$$

I have that:

$$\frac{\partial \dot{x}_t}{\partial x_t} = -\frac{1}{c}\left((\gamma x_t^* + 2\delta)\frac{\gamma c x_t^* - \gamma c x_t^* - c(\gamma x_t^* + \delta)}{\gamma x_t^* + \delta}\right) = (\gamma x_t^* + 2\delta)$$
$$\frac{\partial \dot{x}_t}{\partial f_t} = -\frac{1}{c}\left(-\beta + \gamma\delta\lambda_t^* + (\gamma x_t^* + 2\delta)\left(\frac{\gamma\beta f_t^*}{\gamma x_t^* + \delta} - \frac{\gamma\delta}{\gamma x_t^* + \delta}\lambda_t^*\right)\right)$$
$$= -\frac{\beta}{c}\left(\frac{\gamma x_t^*(\gamma x_t^* + 3\delta)}{\delta(\gamma x_t^* + 2\delta)} - 1\right)$$

As such,

$$J^* = \begin{pmatrix} (\gamma x_t^* + 2\delta) & -\frac{\beta}{c}\left(\frac{\gamma x_t^*(\gamma x_t^* + 3\delta)}{\delta(\gamma x_t^* + 2\delta)} - 1\right) \\ 1 & -\delta \end{pmatrix}$$

I can determine the stability of the steady state by looking at the value of the eigenvalues of J^*. These are given by:

$$\nu_{1,2} = \frac{tr(J^*) \pm \sqrt{tr(J^*)^2 - 4\nabla(J^*)}}{2}$$
$$= \frac{(\gamma \cdot x^* + \delta) \pm \sqrt{(\gamma \cdot x^* + \delta)^2 + 4(\gamma x^* + 2\delta)\delta - 4\frac{\beta}{c}\left(\frac{\gamma x_t^*(\gamma x_t^* + 3\delta)}{\delta(\gamma x_t^* + 2\delta)} - 1\right)}}{2}.$$

Where $\nabla(J^*)$ is the determinant of the Jacobian matrix J evaluated at the steady-state. The largest eigenvalue will always be positive. The smallest eigenvalue will also be positive if $\nabla(J^*) \geq 0$. Let me first focus on the case where $\beta > \delta^2 c$. Developing the inequality yields the following system:

$$x^2 + \frac{2\delta}{\gamma}\frac{\beta - 2\delta^2 c}{\beta - \delta^2 c}x - 2\frac{\delta^2}{\gamma^2}\frac{\beta + 2\delta^2 c}{\beta - \delta^2 c} \geq 0. \tag{C.4}$$

Where the constant of the polynomial is negative and the coefficient associate to the highest coefficient is positive. As such, the polynomial will have a unique positive solution $\hat{x}$, above which the determinant is positive and where the system will be unstable. On the other hand, if $\beta > \delta^2 c$, the inequality in (C.4) switches. In that case, it is easy to see that the constant and the coefficients for the linear and quadratic term are positive. Given that the expression must be lower than 0, it is never satisfied, and the system is always stable.

D Non-monotonic reactions to frustrating events

In real-life scenarios, several inter-temporal dynamics, such as Bayesian updating or learning-by-doing often accompany successive failures. As such, if one wants to look at the effect of frustration accumulation on personal motivation, one might look at a mix of different dynamics. This section studies the interaction one can get when several dynamics are present. In particular, I study the combined effect of learning-by-doing and frustration accumulation on investment and characterise when non-monotonicities of the optimal path can arise.

With that in mind, let me introduce a learning-by-doing mechanism in our system. Let w_t represent the stock of experience accumulated at time t. I assume that w_t follows a logistic growth: $\dot{w}_t = w_t(a - bw_t)$ where $a > b > 0$. The experience growth rate is slow when the decision-maker discovers the task, then increases when she tries to solve it and finally decreases as she perfects her solving mechanism. I suppose that the discount rate of frustration δ is not too large relative to the effect of a trial on experience $\delta < a$. Notice that experience only depends on time and not the agent's investment.[2] The expected instantaneous utility of the agent is:

$$U(x_t, f_t, w_t) = \pi u(x_t, f_t) - v(f_t) - c(w_t, x_t)$$

Learning by doing reduces the marginal cost of investment: $c_{xw}(x, w) < 0$ and has decreasing returns $c_{ww} > 0$, or alternatively $U_{ww}(x, f, w) < 0$. Finally, I focus on the case with negative appraisal effects $U_{xf} \leq 0$, which is the most interesting case.[3]

The inter-temporal optimisation problem is

$$\begin{aligned} V(t_0, f_{t_0}, w_{t_0}) &= \max_{x_t} \int_0^\infty e^{-(\rho+\pi)t} U(x_t, f_t, w_t) dt \\ \dot{f}_t &= x_t - \delta f_t \\ \dot{w}_t &= w_t(a - b w_t) \\ f_{t_0}&, w_{t_0} \text{ given} \end{aligned}$$

I apply the same regularity conditions as before.[4] Let $X(t)$ be the optimal investment path functional solving this maximisation problem. The law of motion of experience is not affected by frustration and investment; it evolves independently.

The system optimisation follows the same lines as before. I can similarly characterise the canonical system and linearise it around the steady-state. This yields the following three-dimensional system. As before, one law of motion describes the evolution of the optimal investment, and the two other reiterates the law of motions of the two-state variables, frustration $\hat{f}_t$ and experience $\hat{w}_t$.

$$\dot{\hat{x}}_t = (\rho + \pi + \delta)\hat{x}_t + \Omega_A^* \hat{f} + \Omega_w^* \hat{w}_t \tag{D.1a}$$

$$\dot{\hat{f}}_t = \hat{x}_t - \delta \hat{f}_t \tag{D.1b}$$

$$\dot{\hat{w}}_t = -a\hat{w}_t \tag{D.1c}$$

[2]It is possible to develop the model with a learning-by-doing process that depends on investment. However, this involves solving a 3^{rd} degree polynomial to get the eigenvalues of the Jacobian, which greatly complicates any further analysis.

[3]Or at least, a case where non-monotonicities arise

[4]That is, the Inada conditions and concavity of the Hamiltonian.

Where Ω_A^* is define as in (6), $\Omega_w^* = \frac{(\rho+\pi+\delta+a)U_{xw}^*}{U_{xx}^*} < 0$ characterise the temporal complementarity between $\hat{x}_t$ and $\hat{w}_t$ in the same way Ω_A^* does for $\hat{x}_t$ and $\hat{f}_t$. The main difference with the system developed in section 3.1, besides the new learning-by-doing law of motion (D.1c), resides in this added term to the investment Euler equation (D.1a). Since $\Omega_w^* < 0$, any additional experience below the steady-state level $\hat{w}_t < 0$, increases the investment level. For example, a $\hat{w}_t$ of one unit below the steady-state, increases the investment level by Ω_w^*

In order to get a picture of how the dynamics play out, let me study the system using the phase diagrams in Figure D.0.1. We are looking for conditions characterising non-monotonicities of the optimal investment path, that is, the cases where $X'(t)$ changes sign. The learning-by-doing effect is represented by a shift of the $\dot{\hat{x}}_t(w) = 0$ locus to the right, indicated by the thick dashed arrow on the graph, from $\dot{\hat{x}}_t(w_{t_0}) = 0$, to $\dot{\hat{x}}_t(w^*) = 0$, its steady-state position.

Let me define the two zones, **I** (**I**ncreasing investment) and **D** (**D**ecreasing investment) in Figure D.0.1. Zone **D** is the area above the $\dot{\hat{f}}_t = 0$ and below the $\dot{\hat{x}}_t(w_t) = 0$ loci, where investment decreases and frustration increases. Zone **I** is the area above the $\dot{\hat{f}}_t = 0$ locus and between $\dot{\hat{x}}_t(w_t) = 0$'s current position and its steady state position. Both investment and frustration increase in zone **I**. Notice that zone **I** is gradually replaced by zone **D** because the investment locus shifts to the right to its steady state position. So a point that is initially in zone **I** would end up in Zone **D** when the $\dot{\hat{x}}_t(w_{t_0}) = 0$ locus reaches its steady state position. This means that the sign of dynamics characterising the evolution of x_t might change when experience increases. Alternatively, their dynamic would change if the optimal investment at time t goes from one zone to another at $t' > t$.

Of course, such two-dimensional phase diagram analysis only partially shows how the system evolves because it cannot represent the dynamics' temporal speed. For example, suppose the initial values of the system are in zone **I**. In that case, will the optimal investment increase until it reaches the steady-state as in Casee 1 depicted in FigureD.0.1? Or will it first increase and then decrease, as depicted in Case 1 of figure D.0.1. Understanding the behaviour of the optimal path $X(t)$ through time ultimately boils down to figuring out $\hat{f}_t$ and $\hat{w}_t$'s relative convergence speed and their relative effect on $\hat{x}_t$. Overall, all the different temporal behaviours of the system can be predicted by looking at whether the initial experience-frustration ratio $\frac{\hat{f}_0}{\hat{w}_0}$ is above or below a certain threshold Θ.[5] I focus on the "*hump-shaped*" investment path thereafter.

[5] See appendix for the definition of Θ.

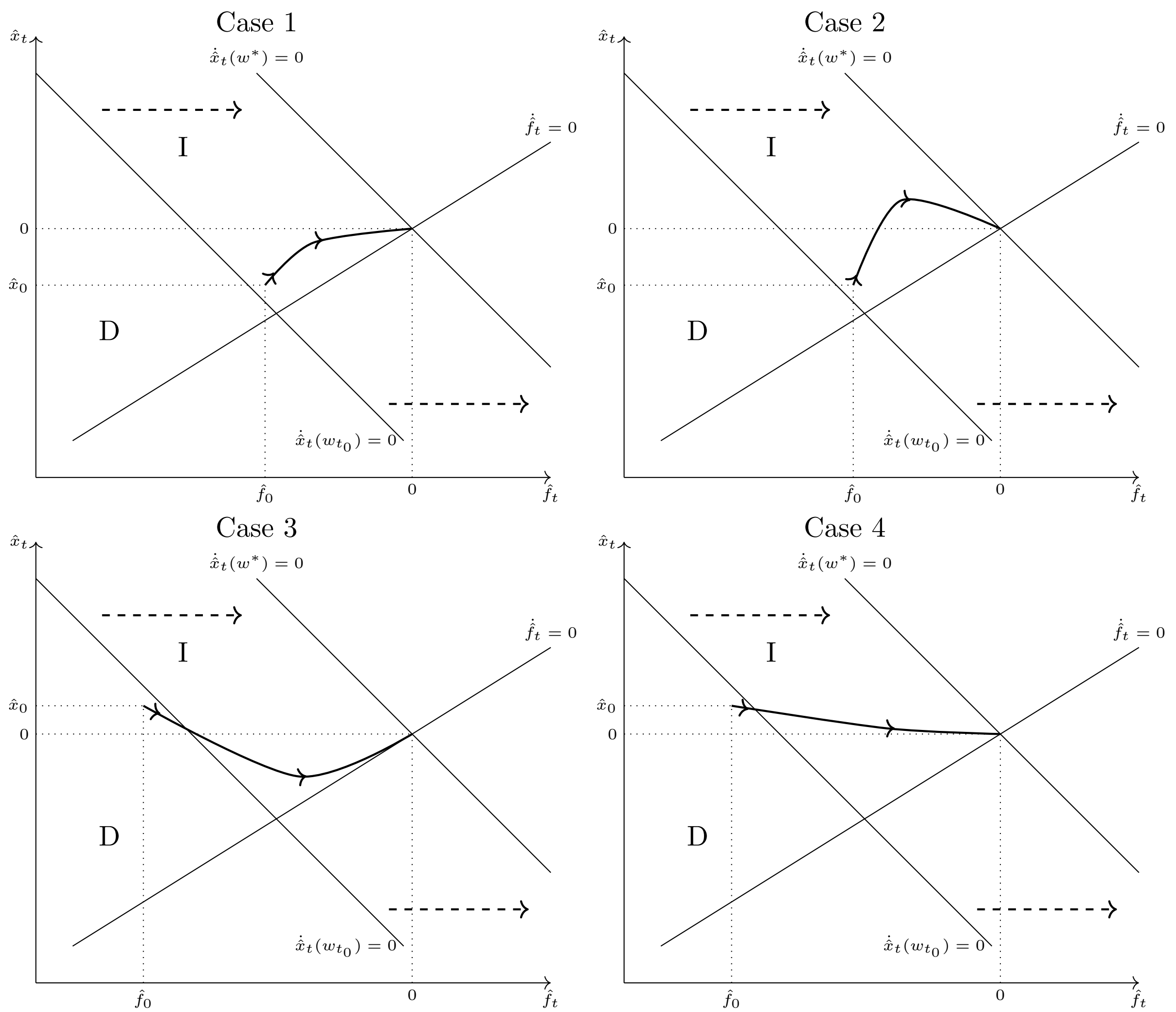


Figure D.0.1: Examples of non-monotonic optimal investment paths in frustration-investment phase diagrams.

Let the frustration-experience ratio be low, with $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$. This can be interpreted as experience being "far enough" from its steady-state value, relative to frustration, given both state variables' effect and convergence speed. For these starting values, $(\hat{x}_0, \hat{f}_0)$ will be in zone **I**, with $\hat{x}_0 \in X(0)$. Since the point is above the $\dot{\hat{x}}_t(w_{t_0}) = 0$ and $\dot{\hat{f}}_t = 0$ loci, frustration and investment increase. But, remember that zone **I** is gradually replaced by zone **D**. As such, there are two possibilities:

(1) If Ω_A^* is low[6], $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, learning-by-doing's effect dominates all the way and frustration's negative effect is never revealed through the dynamics. In that case, the optimal investment $X(t)$ stays in zone **I** as in Case 4 in Figure D.0.1.

(2) If Ω_A^* is large, $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, frustration's effect dominates in the long run but not in the short run, and the optimal investment path is hump-shaped. Intuitively, the task is relatively easy to grapple with, and there is a lot to learn: the learning process is rapid and decreases the marginal cost relatively quickly and significantly. However, once most of the learning occurred, the negative effect of frustration catches up, and investment decreases to its steady-state value. Graphically, the optimal investment level was in zone **I** before a specific $t' > 0$ and is in zone **D** afterwards, yielding a humped-shaped optimal investment path $X(t)$, **I** as in Case 2 in Figure D.0.1.

A final and perhaps more surprising case happens when the effect of frustration on investment is moderate $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, but the initial value of frustration is far from its steady-state value relative to $\hat{w}_0$. Investment first decreases quickly because of the emotional cost but is caught up by the learning mechanism. In this case, the optimal investment path is U-shaped. At first, the agent lowers her investment level because of the overwhelming frustration effect and the slow learning-by-doing mechanism. However, with time, the accumulation of experience becomes dominant relative to frustration, and investment provision increases to its steady-state level. The following proposition summarises the different cases.

Proposition D.1. *Let f_{t_0} and w_{t_0} be below their steady-state values, then:*

- *if $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$ the optimal investment path $X(t)$ is monotone decreasing in time,*
- *if $\Omega_A^* > -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, and $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ the optimal investment path $X(t)$ is first increasing and then decreasing in time,*

[6] Remember that $\Omega_A^* > 0$ with negative appraisal tendencies.

- *if* $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, *and* $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ *the optimal investment path* $X(t)$ *is monotone increasing in time,*
- *if* $\Omega_A^* < -\delta(\rho+\pi+\delta) - a(\rho+\pi+a)$, *and* $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$ *the optimal investment path* $X(t)$ *is first decreasing and then increasing in time.*

D.1 Proof of Proposition D.1

Let $r = \rho + \pi$. The current value Hamiltonian of this problem is:

$$H(x_t, f_t, w_t, \lambda_t,) = U(x_t, f_t, w_t) - \lambda_t(x_t - \delta f_t)$$

Note that I do not include the Lagrangian associated with the learning-by-doing law of motion as its Langrangian multiplier does not impact the control variable and state variables' differential equation. This simplifies the exposition of the result. Solving the maximisation problem on the state-costate space with four differential equations (two states, two costates) naturally yields the same result. The first-order conditions of the system are

$$\begin{cases} U_x(x_t, f_t, w_t) = \lambda_t \\ \dot{\lambda}_t = U_f(x_t, f_t, w_t) + (r+\delta)\lambda_t \end{cases} \tag{D.2}$$

Differentiating the first equation of (D.2) with regards to time and inserting the law of motion of the costate yields the canonical system:

$$\begin{cases} \dot{x}_t &= \dfrac{1}{U_{xx}(x_t, f_t, w_t)}\Big((\dot{\lambda}_t - \dot{w}_t \cdot U_{xw}(x_t, f_t, w_t) - \dot{f}_t \cdot U_{xf}(x_t, f_t, w_t)\Big) \\ \dot{f}_t &= x_t^* - \delta f_t \\ \dot{w}_t &= w_t(a - bw_t) \end{cases}$$

The Jacobian of the system at the steady-state is:

$$J(x^*, f^*, w^*) = \left.\begin{pmatrix} \frac{\partial \dot{x}(x,f,w)}{\partial x} & \frac{\partial \dot{x}(x,f,w)}{\partial f} & (\frac{\partial \dot{x}(x,f,w)}{\partial w} \\ \frac{\partial \dot{f}(x,f,w)}{\partial x} & \frac{\partial \dot{f}(x,f,w)}{\partial f} & \frac{\partial \dot{f}(x,f,w)}{\partial w} \\ \frac{\partial \dot{w}(x,f,w)}{\partial x} & \frac{\partial \dot{w}(x,f,w)}{\partial f} & \frac{\partial \dot{w}(x,f,w)}{\partial w} \end{pmatrix}\right|_{(x,f,w)=(x^*,f^*,w^*)}$$

$$= \begin{pmatrix} r+\delta & \Omega_f^* & (r+\delta+a)\frac{U_{xw}^*}{U_{xx}^*} \\ 1 & -\delta & 0 \\ 0 & 0 & -a \end{pmatrix}.$$

The eigenvalues of this system are μ_1 and μ_2, defined as in Section A.2.1 and $\mu_3 = -a$. Let me define wlog that $\mu_1 < \mu_2$.

I can now compute the eigenvector $u_1 = (u_1^1, u_1^2, u_1^3)'$ and $u_3 = (u_3^1, u_3^2, u_3^3)'$ associated to the negative eigenvalues μ_1 and μ_3. These are found by solving the system $(J^* - \mu_i \times I) * u_i = 0$, where I is the 3 by 3 identity matrix, $i = 1, 3$:

$$\begin{cases} (r+\delta-\mu_i)u_i^1 + \Omega_f^* u_i^2 + (r+\delta+a)\frac{U_{xw}^*}{U_{xx}^*}u_i^3 = 0 \\ u_i^1 + (-\delta-\mu_i)u_i^2 = 0 \\ (-a-\mu_i)u_i^3 = 0 \end{cases} \tag{D.3}$$

For μ_1, given the third row of the system, $u_1^3 = 0$ since $\mu_1 \neq -a$. Using the second row, it is easy to find that $u_1^2 = \frac{\mu_1^1}{\delta+\mu_1}$. As such, $(u_1^1, u_1^2, u_1^3) = (1, \frac{1}{\delta+\mu_1}, 0)$ is one of the solutions of the system.

Next, for μ_3, normalise u_3^1 to 1 and use the second row of the system to get $u_3^2 = \frac{1}{\delta+\mu_3}$.

As for u_3^3, I can plug the previous result into the first row to get:

$$
\begin{aligned}
-u_3^3(r+\delta-\mu_3)\frac{U_{xw}^*}{U_{xx}^*} &= \left((r+\delta-\mu_3)+\frac{\Omega_f^*}{\delta+\mu_3}\right)u_3^1 \\
&= \left(\frac{(r+\delta-\mu_3)(\delta+\mu_3)+\Omega_f^*}{\delta+\mu_3}\right) \\
&= \frac{1}{\delta+\mu_3}\left(\Omega_f^*+\delta(r+\delta)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(\frac{1}{4}\left(r^2-r^2+4(\Omega_f^*+\delta(r+\delta))\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(\frac{1}{4}\left(\left(\sqrt{r^2+4(\Omega_f^*+\delta(r+\delta))}\right)^2-r^2\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}\left(-\mu_1\left(r-r+\frac{r+\sqrt{r^2+4(\Omega_f^*+\delta(r+\delta))}}{2}\right)+\mu_3(r-\mu_3)\right) \\
&= \frac{1}{\delta+\mu_3}(\mu_3(r-\mu_3)-\mu_1(r-\mu_1)) \\
&= \frac{1}{\delta+\mu_3}(\mu_3-\mu_1)(r-\mu_3-\mu_1) \\
\iff \quad u_3^3 &= \frac{U_{xx}^*}{U_{xw}^*}\frac{(\mu_1-\mu_3)(r-\mu_3-\mu_1)}{(r+\delta-\mu_3)(\delta+\mu_3)}
\end{aligned}
$$

Given the eigenvalues and associated eigenvectors, I can express the solution of our system as:

$$
\begin{cases}
\hat{x}_t &= k_1e^{\mu_1 t}+k_3e^{\mu_3 t} \\
\hat{f}_t &= k_1u_1^2e^{\mu_1 t}+k_3u_3^2e^{\mu_3 t} \\
\hat{w}_t &= k_3u_3^3e^{\mu_3 t}
\end{cases}
\tag{D.4}
$$

Where the initial values of the system determine k_1 and k_3. For what follows, it is important to note that I have $\delta+\mu_3<0$ by assumption and since and $\delta+\mu_1<0$ since $\Omega_A^*>0$ [7], as we are considering negative appraisal tendencies. Next, notice that:

$$
\hat{f}_0<0 \iff k_1>-k_3\frac{\mu_1+\delta}{\mu_3+\delta}
\tag{D.5}
$$

The following result gives conditions for system (D.4)'s investment path $\hat{x}_t$ to be non-monotonic in time:

[7] As shown Appendix A.2.1

Lemma D.1. *A system defined as in* (D.4) *with two negative eigenvalues* μ_1 *and* μ_3*, exhibits a non-monotonic optimal investment path* $\hat{x}_t$ *with regards to time if and only if* k_1 *and* k_3 *have opposite signs and either* $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ *and* $\mu_3 < \mu_1$ *or* $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$ *and* $\mu_3 > \mu_1$*. Moreover,* $\dot{\hat{x}}_t$ *only changes sign once.*

Proof. A necessary condition for the optimal investment path to be non-monotonic is that at some t', $\dot{\hat{x}}_{t'} = k_1\mu_1 e^{\mu_1 t} + k_3\mu_3 e^{\mu_3 t} = 0$. This happens if and only if there is a positive t' solving:

$$\ln\left(-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3}\right)\frac{1}{\mu_3 - \mu_1} = t'$$

Since μ_1 and μ_3 are negative, this equation is well defined if k_1 and k_3 have opposite signs. Moreover, t' will be positive if either $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ and $\mu_3 < \mu_1$ or $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$ and $\mu_3 > \mu_1$. Notice that there is only one t' satisfying this equation. As such, if t' characterises an extremum, it is unique.

For sufficiency, I need to verify that t' is an extremum, not an inflexion point. This can be easily checked by looking at the sign of the second derivative of $\hat{x}_t$

$$\ddot{\hat{x}}_t = \mu_1^2 k_1 e^{\mu_1 t} + \mu_3^2 k_3 e^{\mu_1 t}$$

$\hat{x}_t$ will change its curvature at the possible inflexion point t'' if $\ddot{\hat{x}}_{t''} = 0$. If t'' exists, it is defined by:

$$t'' = \ln\left(-2\frac{k_1}{k_3}\frac{\mu_1}{\mu_3}\right)\frac{1}{\mu_3 - \mu_1} > t'$$

As such, t' is not an inflexion point.

□

Let me now prove the result of the proposition. First, as shown before, I have that:

$$u_3^3 = \frac{U^*_{xx}}{U^*_{xw}} \cdot \frac{-\Omega^*_f - \delta(r+\delta) - a(r+a)}{(\delta+\mu_3)(r+\delta-\mu_3)} \tag{D.6}$$

$$= \frac{U^*_{xx}}{U^*_{xw}} \cdot \frac{(\mu_1 - \mu_3)(r - \mu_1 - \mu_3)}{(\delta+\mu_3)(r+\delta-\mu_3)} \tag{D.7}$$

I can see by inspecting (D.6)-(D.7) that $-\Omega^*_f - \delta(r+\delta) - a(r+a)$ has the same sign as $(\mu_1 - \mu_3)$. Next, let me define the threshold of Θ of the proposition as:

$$\Theta = \frac{U^*_{xw}}{U^*_{xx}} \cdot \frac{(\delta + \mu_1 + \mu_3)(r + \delta - \mu_3)}{\mu_1(\delta + \mu_1)(r - \mu_1 - \mu_3)} > 0 \tag{D.8}$$

I can, therefore express u_3^3 in the following way:

$$u_3^3 = \frac{1}{\Theta} \frac{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)}{\mu_1(\delta + \mu_1)(\delta + \mu_3)}$$

It is possible to determine the expression of k_1 and k_3 by setting $t = 0$:

$$k_3 = \frac{w_{t_0}}{u_3^3} = \frac{\mu_1(\delta + \mu_1)(\delta + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \tag{D.9}$$

$$k_1 = \frac{1}{u_1^2} \left(f_{t_0} - u_3^2 \cdot k_3 \right) = (\delta + \mu_1) \left(f_{t_0} - \frac{\mu_1(\delta + \mu_1)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \right) \tag{D.10}$$

Next, notice that if $f_{t_0} \geq \Theta w_{t_0}$, (and similarly for $f_{t_0} < \Theta w_{t_0}$):

$$\begin{aligned}
& f_{t_0} \geq \Theta w_{t_0} \\
\iff & f_{t_0} \geq \frac{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \\
\iff & (\delta + \mu_1) \left(f_{t_0} - \frac{\mu_1(\delta + \mu_1)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \right) \leq - \frac{\mu_3(\delta + \mu_1)(\delta + \mu_3)}{(\mu_1 - \mu_3)(\delta + \mu_1 + \mu_3)} \Theta w_{t_0} \\
\iff & k_1 \leq -k_3 \frac{\mu_3}{\mu_1}
\end{aligned}$$

The inequality switches on the third line because $\delta + \mu_1 < 0$ and where k_1 and k_2 are inserted in the expression using equations (D.10) and (D.9). As such:

$$\frac{f_{t_0}}{w_{t_0}} \leq (>) \Theta \iff k_1 \leq (>) - k_3 \frac{\mu_3}{\mu_1} \tag{D.11}$$

I can now use the equivalence relation (D.11) and Lemma D.1 to determine when non-monotonicities arise.

- If $\Omega_f^* > -\delta(r + \delta) - a(r + a)$, by expression (D.6)-(D.7), I have that $\mu_3 > \mu_1$ and $u_3^3 < 0$. Since $w_{t_0} < 0$, it must be that $k_3 > 0$ because $w_{t_0} = k_3 u_3^3$. Now given that $f_{t_0} < 0$, there are two cases:

  1. $\frac{\hat{f}_0}{\hat{w}_0} < \Theta$ from expressions (D.11) and (D.5), k_1 is in $\left(-k_3 \frac{\mu_1 + \delta}{\mu_3 + \delta}, -k_3 \frac{\mu_3}{\mu_1} \right)$, and

$k_1 < 0$. Then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} > 1$. By Lemma D.1, the optimal path is non-monotonic. I have that:

$$\dot{\hat{x}}_0 = \mu_1 k_1 + \mu_3 k_3 > -k_3 \frac{\mu_3}{\mu_1}\mu_1 + \mu_3 k_3 = 0$$

Also, by Lemma D.1, $\dot{\hat{x}}_t$ only changes sign once. As such, the optimal investment path is first increasing and then decreasing.

2. $\frac{\hat{f}_0}{\hat{w}_0} \geq \Theta$, from expression (D.11), $k_1 \geq -k_3\frac{\mu_3}{\mu_1}$,then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} \leq 1$ .By Lemma 2, the optimal path is monotone Since $\dot{x}_0 = k_1\mu_1 + k_3\mu_3 \leq 0$, the optimal path is monotone decreasing.

- If $\Omega_f^* < -\delta(r-\delta) - a(r+a)$ then, $\mu_1 > \mu_3$, $u_3^3 > 0$ and $k_3 < 0$. I then have two possibilities:

1. $\frac{\hat{f}_0}{\hat{w}_0} \leq \Theta$, as such, $k_1 \in \left(-k_3\frac{\mu_1+\delta}{\mu_3+\delta}, -k_3\frac{\mu_3}{\mu_1}\right]$, then $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} \leq 1$. By Lemma D.1, the optimal path is monotonic. Since $\dot{x}_0 \geq 0$. It is first monotone increasing.
2. $\frac{\hat{f}_0}{\hat{w}_0} > \Theta$, as such, $k_1 > -k_3\frac{\mu_3}{\mu_1}$ then $\dot{x}_0 < 0$ and $-\frac{k_1}{k_3}\frac{\mu_1}{\mu_3} < 1$ and $\mu_1 > \mu_3$, by Lemma D.1, the optimal path is non-monotonic: it is first decreasing and then increasing.

E The psychology behind the model

This section links the modelling choices to the psychological literature for more specialised readers.

It might be first important to repeat that frustration should not be understood as it does in its ordinary meaning as a single discrete emotion. One can consider frustration as some proto-emotion that stems from the primary appraisal of the frustrating events as relevant and goal incongruent (Lazarus and Folkman, 1984).

Frustration should also not be considered as a mood. First, frustration stems from frustrating events, which are a particular cause, while moods are less focused Keltner and Lerner (2010). Second, frustration is context-bound and interacts with the decision environment and the state of the world through the emotional relief effect in a way a mood would not. Although emotions are thought to be short-lived, which is modelled through the decay rate, the model also shows how an emotion can persist for a longer time if it triggers an action whose consequences trigger the emotion itself.[8]

Even though the title of this article might indicate that I am following a valence approach to emotion, I am not. While it is true that the emotional cost function plays an essential role in how emotions affect behaviour, it is only part of the picture. When looking at the model's solution and the impact of frustration on investment provision, it becomes clear that the behavioural reaction to frustration - the *action tendency* - will depend on much more than valence. In particular, many determinants of the optimal solution map directly to dimensions identified in dimensional approaches to cognitive appraisal, especially Smith and Ellsworth (1985). These include certainty (the level of $\bar{\pi}(x_t)$), control coping (through γ), pleasantness (the amount of frustration triggered by a failure and its marginal cost) and anticipated effort through the expectation of future effort cost and frustration cost in case of continuing failure. In fact, following a dimensional approach to emotions, Appendix D, shows that one can have a transition of action tendencies when the appraisal of one dimension changes (a change in the anticipated effort cost through the learning-by-doing process).

Finally, the model also adopts the Appraisal-Tendency framework (Lerner and Keltner, 2001), a cognitive appraisal model with a dimensional approach to appraisal. It should be noted that while keeping the model general by not describing the context in which

[8]This mechanism is central in recent development in psychopathology (Robinaugh et al., 2020). It might, therefore, not be surprising that one can transform the framework into a rational addiction model, see Appendix C.

frustration emerges, one also loses some precision about how appraisal tendencies work. In particular, Appraisal Tendencies are related to the core appraisal theme of the emotion triggered. Here, appraisal tendencies come into a more reduced form fashion as I do not relate the appraisal of the environment to subsequent appraisal tendencies. In fact, Appraisal tendencies could (and should) be endogenised when studying more specific situations. However, approaching the question in a more general reduced form fashion also has its advantages. First, it gives a general benchmark of how appraisal tendencies, wherever they come from, can influence behaviour. Second, it provides a theoretical benchmark one can use to study appraisal tendencies when researchers have limited information about the emotional process, as in Section 4.

F Empirical Appendix

F.1 Pitch Type analysis

Table F.1.1 presents the standard speed variation for all pitches representing at least one per cent of the cleaned sample. The standard deviations are computed for each pitch type at the player-game-inning level. Table F.1.1 presents these standard deviations' averages over the entire sample for each pitch type. The standard deviations are remarkably homogeneous and vary from 0.91 to 1.08 mph.

Table F.1.1: Standard deviation of Speed of pitches

Pitch Name	SD Speed	Number of Obs
Sinker	0.91	1549913
Slider	1.08	1103659
4-Seam Fastball	0.92	2505992
Split-Finger	0.96	110276
Cutter	0.94	412864
Changeup	0.95	742088
Curveball	1.09	587691
Knuckle Curve	1.06	151197

The first column indicates the pitch type. I only keep pitches that represent more than 1% of the observation of the dataset. The second column indicates the average standard deviation of the speed of a pitch type during a *game* × *inning* at the player level. The third column indicates the number of throws for each pitch type in the cleaned dataset.

Table F.1.2 presents the frustration coefficients for the most popular pitches when running the OLS regression, controlling for all factors presented in Table 3. The effect of frustration on speed is robust whether you look at the effect at the pitch type level or the aggregate level. Frustration seems to affect some pitch types more than others, with a higher effect on Fastballs and a lower effect on Split Fingers.

Table F.1.2: Effect of frustration per pitch type

Dependent Variable:	ln Velocity	
Model:	(1)	(2)
Variables		
Fastball	0.0017***	0.0017***
	(5.8×10^{-5})	(4.1×10^{-5})
Split Finger	0.0009**	0.0009***
	(0.0004)	(0.0003)
Slider	0.0014***	0.0014***
	(0.0001)	(8.59×10^{-5})
Curveball	0.0012***	0.0012***
	(0.0002)	(0.0001)
Cutter	0.0012***	0.0012***
	(0.0001)	(0.0001)
Changeup	0.0008***	0.0008***
	(0.0001)	(9.11×10^{-5})
Knuckle Curve	0.0017***	0.0017***
	(0.0002)	(0.0002)

The table's first column indicates which pitch type the regression was performed. The columns indicated by (1) and (2) display the OLS coefficient for frustration when the log of pitch speed is the dependent variable, controlling for all fixed effects present in Table 3. Column (1) is clustered at the pitcher level, and column (2) is clustered at the game level.

F.2 Frustration effect on pitch quality

The pitch quality measure is the result of an estimation predicting pitches' success, given the physical characteristics of the motion of the throw. I use the eXtreme Gradient Boosting (XGBoost) algorithm (Chen and Guestrin, 2016) to do this. XGBoost is a gradient-boosting model based on decision tree ensembles. The tree ensemble is a set of classification trees where each leaf is assigned a prediction score. XGBoost then aggregates the prediction of all the trees to get the final prediction. The main difference with random forest models, which are also based on tree ensembles, is how each tree is optimised.

New trees aim to minimise the residual errors in the predictions from the existing sequence of trees. Each tree is sequentially (instead of simultaneously, as in Random Forests algorithms) learned by optimising a binary logistic objective function. I use the 2020-2021 seasons as a training dataset. I set the number of trees to 100. I tuned the algorithm's hyper-parameters using a random grid search and a 5-fold cross-validation. I focus on two hyperparameters. The first is the learning rate $\eta \in (0, 1]$, which reduces the weight of new features in the predictions to avoid over-fitting. The second controls the L2 regularisation term. Table F.2.1 describes the features used in the machine learning algorithm. I train a separate algorithm for each pitch for which I have more than 10,000 observations in the training dataset.

Table F.2.1: Features used in the estimation of the Success Probability

Features
Velocity of the pitch, in x, y and z-dimension, at y=50 feet.
The acceleration of the pitch in x, y and z-dimension, determined at y=50 feet.
Horizontal & vertical ball position when it crosses home plate (catcher's perspective).
Horizontal & vertical movement in feet from the catcher's perspective.
Out-of-hand pitch velocities.
Horizontal & vertical Release Position of the ball measured in feet(catcher's perspective).
Release position of pitch measured in feet (catcher's perspective).
Acceleration of the pitch, in feet per second per second, in x, y and z-dimension.
Top & bottom of the batter's strike zone when the ball is halfway to the plate.

Table F.2.2 show the result of the machine learning algorithm when applied to the main dataset (2010-2019). I measure the accuracy as the proportion of observation in the dataset for which the algorithm predicts a probability of more than 0.5 for the event that actually happened (Success or Failure). The first column shows that the algorithm's accuracy is between 0.81 and 0.75, depending on the pitch type. The next two columns indicate the average probability predicted given the (ex-post) outcome. This shows that the algorithm does a good job of differentiating the outcome.

Table F.2.2: Predictive Statistics of the Machine learning algorithm

Pitch Name	Accuracy	Mean Pred.prob(S|S)	Mean Pred.prob(F|F)
Sinker	0.79	0.75	0.31
Slider	0.76	0.70	0.29
4-Seam Fastball	0.81	0.76	0.27
Split-Finger	0.75	0.68	0.31
Cutter	0.78	0.72	0.30
Changeup	0.76	0.70	0.29
Curveball	0.77	0.72	0.29
Knuckle Curve	0.77	0.69	0.29

The first column indicates the pitch type for which the algorithm ran. The second indicates the proportion of observations for which the predicted probability tilted towards the right pitch outcome. That is the proportion of observation for which the pitches ended up in a failure (or success) and where the predicted probability of success was greater (or lower) than 0.5. The third and fourth columns indicate the average predicted probability of success given the outcome of the pitch ((S)uccess or (F)ailure).

To assess the impact of frustration on quality, I run an OLS regression with the standard control strategy that was carried on in the rest of the paper, using the quality variable derived from the XGBoost estimation as the dependent variable. Table F.2.3 shows that one unit of frustration decreases the predictive probability of success by half a percentage point. Although the effect is significant and economically meaningful, the control strategy is less successful for these regressions. Indeed, the adjusted R^2 is much lower and around 7.7%.

Table F.2.3: Effect of Frustration on Quality

Dependent Variable:	Quality	
Model:	(1)	(2)
Variables		
$\mathbf{F_t}$	-0.0048***	-0.0048***
	(0.0007)	(0.0007)
# Failures	0.0138***	0.0138***
	(0.0004)	(0.0001)
# Successes	-0.0091***	-0.0091***
	(0.0003)	(0.0001)
Attempt	0.0030***	0.0030***
	(0.0002)	(5.19×10^{-5})
Δ Exp.	-0.0252***	-0.0252***
	(0.0012)	(0.0010)
Δ Score	0.0039***	0.0039***
	(0.0003)	(0.0002)
Δ Inning Score	0.0032***	0.0032***
	(0.0011)	(0.0010)
Fixed-effects		
P_{type}	Yes	Yes
Game State	Yes	Yes
Player × Game	Yes	Yes
Batter	Yes	Yes
Inning	Yes	Yes
Fit statistics		
Observations	7,163,516	7,163,516
R^2	0.07660	0.07660
Within R^2	0.00438	0.00438

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Column (1) and (2) are clustered at the pitcher level and game level. $\mathbf{F_t}$ measures the change in the other team's expected score during the current and (possibly empty) sequence of consecutive failure. # Failures and # Successes count the number of failures and success since the beginning of the inning. *Attempt* measures the number of pitches the pitcher has thrown since the beginning of the inning. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ Inning Score measures the change in score since the beginning of the inning. P_{type}indicates the pitch type. Game State indicates the number of batters out, the number of balls, strikes and base occupancy. Player × Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

F.3 Individual level Analysis

Let me introduce an individual-level analysis of pitchers' reactions to frustration. I first select all pitchers for which I have more than 2000 registered pitches, for a total of 859 pitchers[9]. My first exercise is to perform the same regression as in column 1 of table 3 for speed in the main text and F.2.3 in Appendix F.2 for quality. Figure F.3.1 show the distribution of the coefficients. The frustration speed coefficient distribution clearly has a left skew, with most of the mass being at the right of zero. The frustration quality coefficient distribution only shows a very slight skew to the right.

Figure F.3.1: Distribution of the frustration coefficients.

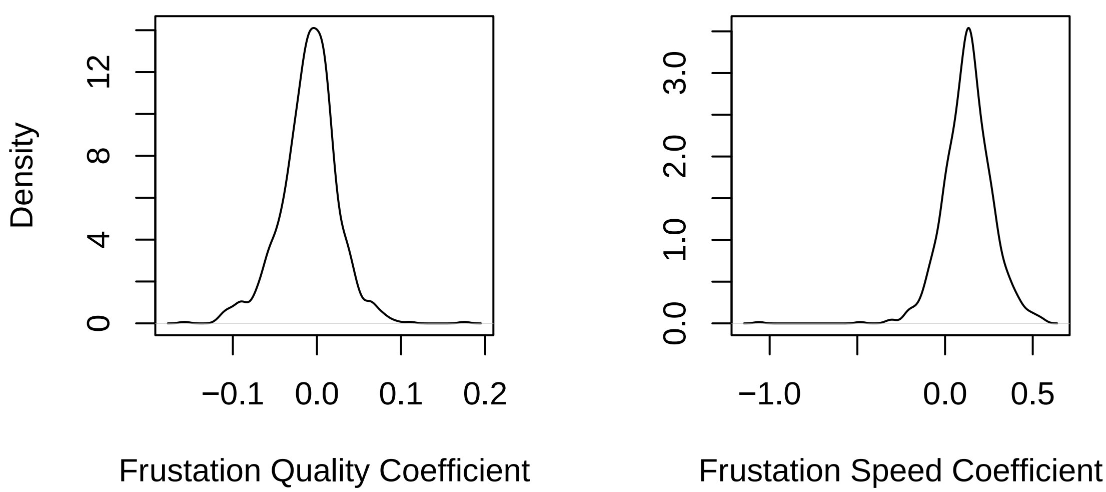


Table F.3.1 shows that more than one-third of the sample show a significant effect of frustration accumulation on their behaviours regarding pitch velocity. Note that frustration accumulation at a high level remains rare. As such, estimating a significant impact on a smaller sample size is much more challenging. The effects are also remarkably homogeneous. 98% of the significant frustration coefficients are positive. Regarding quality, the results are more modest, and only 8.6% of the sample exhibit a significant effect of frustration on the quality of the pitches. For the majority of pitchers, this effect is negative.

[9]This creates some selection bias because I am effectively selecting for the most competitive pitchers in MLB. Table F.3.1 shows the results.

Note that the 9% that have a significant effect of frustration on quality must have a p-value below 5% percent. As such, we are above the threshold for false positives. Together with the false negative in the sample, there must be a small minority of professional pitchers whose frustration impacts quality.

Table F.3.1: Individual-level sensitivity to frustration

	Pos. .Coeff.	Neg. .Coef
Speed	0.38	0.01
Quality	0.02	0.07

The first row represents the proportion of pitchers displaying a significant effect (at at least 5% significance level) of frustration on speed for positive and negative coefficients. The second row displays the same results for the Quality regressions.

Table F.3.2 shows correlations for the individual-level coefficients. There is a significant and negative correlation between the effect of frustration on speed and its impact on pitch quality. A higher effect of frustration on speed is associated with a more negative impact on quality. This rules out any rationale for saying that frustration motivates pitchers to reach a better optimum. A possible rationale is the following. Frustration increases the dis-alignment of emotional and performance incentives. For some players, this is limited to a slight change in speed when frustrated. For others, the effect is such that it decreases the quality of pitches. Overall, this goes towards the appraisal tendencies' interpretation of the results.

Next, I look at whether frustration sensitivity impacts performance at the career level. To do this, I use the xFIP metric, which is short for Expected Fielding Independent Pitching. xFIP measures the pitcher's average performance for events where the pitcher has the most control, such as strikeouts, home runs, walks, or hit by pitches[10]. For these events, his teammate's performance should play no role.

[10]It also corrects for the Home-run-to-fly-ball rate season's league average.

Table F.3.2: Frustration coefficient: correlations

	Speed	Quality
Speed		
Quality	-0.11***	
xFIP	-0.06*	0.11***

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Correlation table between the Speed and Quality Frustration coefficient, and XFIP metric. xFIP is a measure that measures pitchers' skills and focuses on the outcomes over which the pitchers have the most control.

On its own, frustration's impact on speed correlates very weakly with a general performance decrease. However, frustration's influence on behaviour can be high enough to trigger negative consequences on pitch quality. In that case, frustration seems to be associated with a career-level decrease in the pitcher's performance. Although modest, the correlation is significant at a 1% significance level. The fact that the correlation is not higher is not surprising. The frustration coefficient only tells how the quality evolves when the pitcher is frustrated and says nothing about the pitcher's performance when he is not (80% of the throws according to figure 2).

F.4 Robustness check on restricted sample

For some observations, frustration will always be strictly greater than 0. In particular any observation where the pitcher fails consecutively since the beginning of the inning will always have a strictly positive value of frustration. One might argue that there is not enough variation in those observations to differentiate the effect of frustration from the effect of the strategic game state. To account for this, I carry out the same OLS regression without all observations where the agent suffered an unbroken sequence of failed attempts since the beginning of the inning. The deleted observations are all those where there are no strikes and no outs, after the first throw. Table F.4.1 show the results:

Table F.4.1: Effect of Frustration on Velocity with restricted sample

Dependent Variable:	Velocity	
Model:	(1)	(2)
Variables		
$\mathbf{F_t}$	0.1076***	0.1076***
	(0.0051)	(0.0037)
# Failures	0.0186***	0.0186***
	(0.0013)	(0.0008)
# Successes	0.0337***	0.0337***
	(0.0015)	(0.0007)
Attempt	-0.0143***	-0.0143***
	(0.0012)	(0.0004)
Δ Exp.	0.0237***	0.0237***
	(0.0071)	(0.0063)
Δ Score	0.0015	0.0015
	(0.0022)	(0.0012)
Δ Inning Score	-0.0839***	-0.0839***
	(0.0076)	(0.0060)
Fixed-effects		
P_{type}	Yes	Yes
Game State	Yes	Yes
Player $\times$ Game	Yes	Yes
Batter	Yes	Yes
Inning	Yes	Yes
Fit statistics		
Observations	6,654,620	6,654,620
R^2	0.91787	0.91787
Within R^2	0.00226	0.00226

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Column (1) and (2) are clustered at the pitcher level and game level. $\mathbf{F_t}$ measures the change in the other team's expected score during the current and (possibly empty) sequence of consecutive failure. # Failures and # Successes count the number of failures and success since the beginning of the inning. *Attempt* measures the number of pitches the pitcher has thrown since the beginning of the inning. Δ Exp measure the change in the opposing team's expected score since the beginning of the inning. Δ Score measures the current difference in score, while Δ Inning Score measures the change in score since the beginning of the inning. P_{type} indicates the pitch type. Game State indicates the number of batters out, the number of balls, strikes and base occupancy. Player $\times$ Game indicates who is pitching during which game. *Batter* indicates who is currently facing the pitcher.

Table F.4.1 show that the result is robust to the sample manipulation and remains qualitatively similar.

F.5 Model adapted to the application

This section adapts the model's stochastic structure to fit the illustration's environment and result. Given that speed does not affect the probability of success locally, let me consider a fixed arrival rate of success π. There are two major differences with the main specification of the model. (1) the amount of frustration gained from one period to another only depends on the pitch outcome and how it affects score expectation. Since it does not depend on the pitch velocity x, the frustration dynamics will be a frustration accumulation process with exogenous increments as in section B. However, contrary to section B, the amount of frustration gained after a failed attempt is also stochastic, and not simply a deterministic increment $a > 0$. (2) There are also other stochastic components in the game that might affect the decisions and the pitcher's utility.

As such, to consider both (1) and (2) I need to consider a more general stochastic structure. Let $\omega_t \in \Omega \subset \mathbb{R}$ represent the relevant state space. Some dimensions of ω_t will pertain to the frustration level, others to the strategic state of the game. I assume that the stochastic process $\{\omega_t, t \geq 0\}$ has the Markov property. As such, transition densities only depend on the current state ω_t. Note that the pitch velocity variation observed in the data is unlikely to affect the (stochastic) transition from one state to another.[11] This independence is helpful, as it effectively transforms a very complicated dynamic maximisation problem into a static one. To see this, one can set up the general Hamilton-Jacobi-Bellman equation associated with the maximisation problem. I have that:

$$(\rho + \pi)V(f, \omega) = \max_x \{U(x, f, s) + \frac{1}{dt}\mathbb{E}_w V(f, \omega)\} \tag{F.1}$$

The expectation of the value function on the right-hand side does not depend on x, and the first-order condition is:

$$\frac{\partial \mu(x, f)}{\partial x} = c'(x) \tag{F.2}$$

As such, optimal speed does not depend on the value function, and the pitcher only reacts to frustration, as it cannot influence its future value. Using the implicit function theorem on the first-order conditions directly shows that the appraisal tendency would drive the empirical results.

[11] Or, a least marginally enough not to be considered here.

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